Welcome to Negative Numbers!

Etymology: late Middle English: from late Latin *negativus*, from *negare* ‘deny’

What are Negative Numbers?

- numbers less than zero

Where do we use Negative Numbers?

Some examples

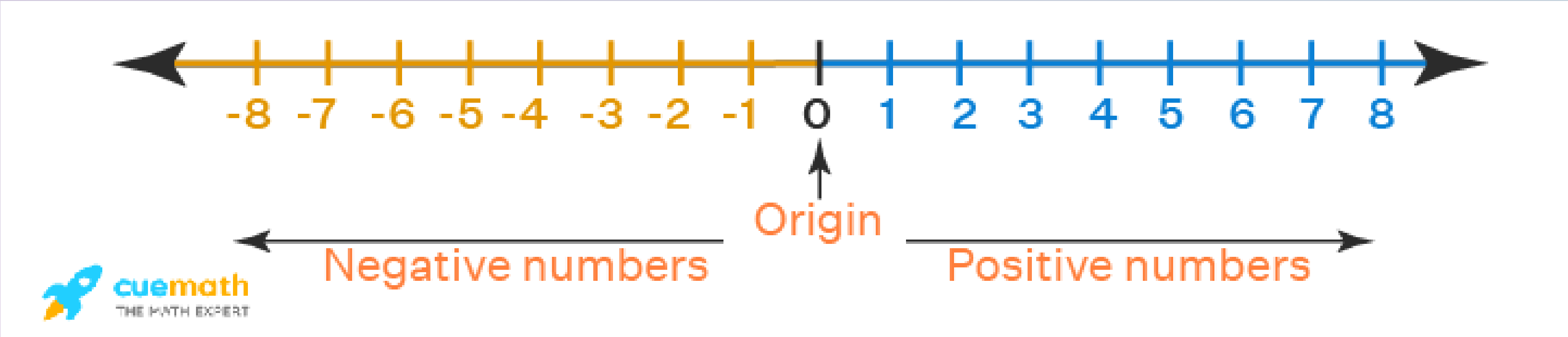
- freezing temperatures
- when you owe money
- elevation below sea level
- penalty points in sports and games

The New Number Line

New half unlocked!

You've heard of whole numbers and fractions, now get ready for:

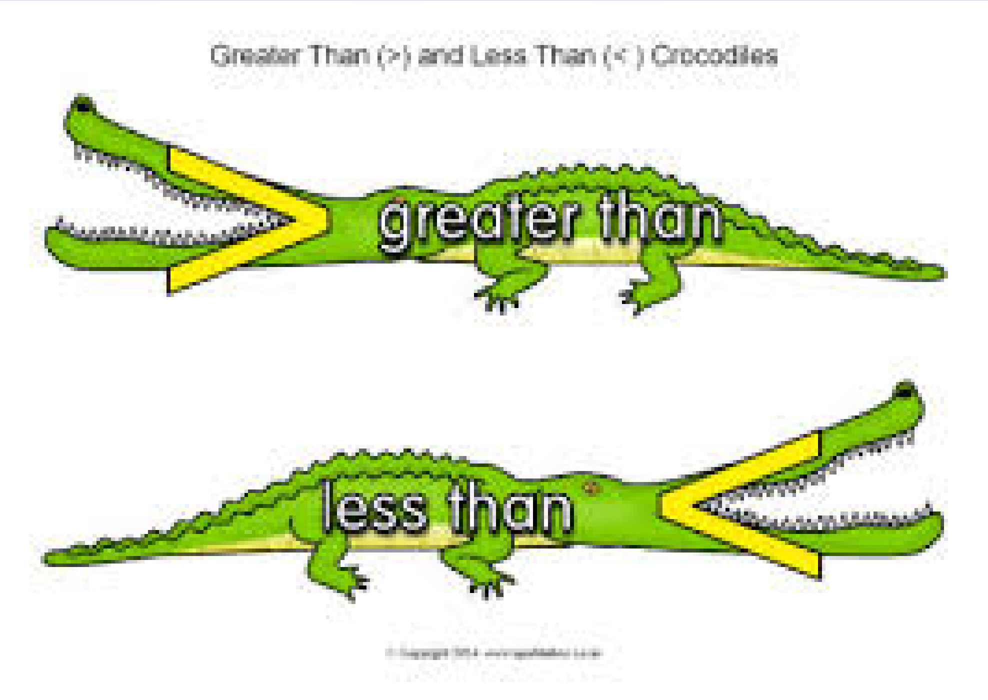
- **Integers**: all the whole negative numbers, whole positive numbers *and* zero, e.g. 5, -4, 0



- Do negative fractions also belong on the number line?
- Why are negative numbers left of zero?
- Is -8 greater than -5?
- Is -8 greater than 5?

Ordering

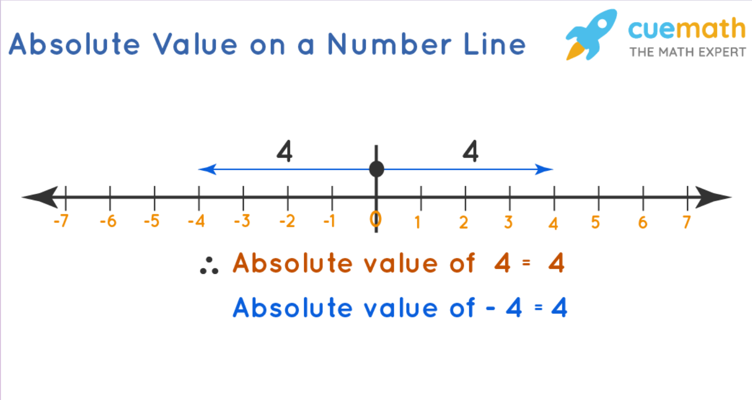
- '>' means 'is greater than', e.g. 5 > 4
- '<' means 'is less than', e.g. 6 < 10
- Ascending: from smallest to largest (going from bottom to top like stairs)
- Descending: from largest to smallest (like coming down the stairs)



Magnitude

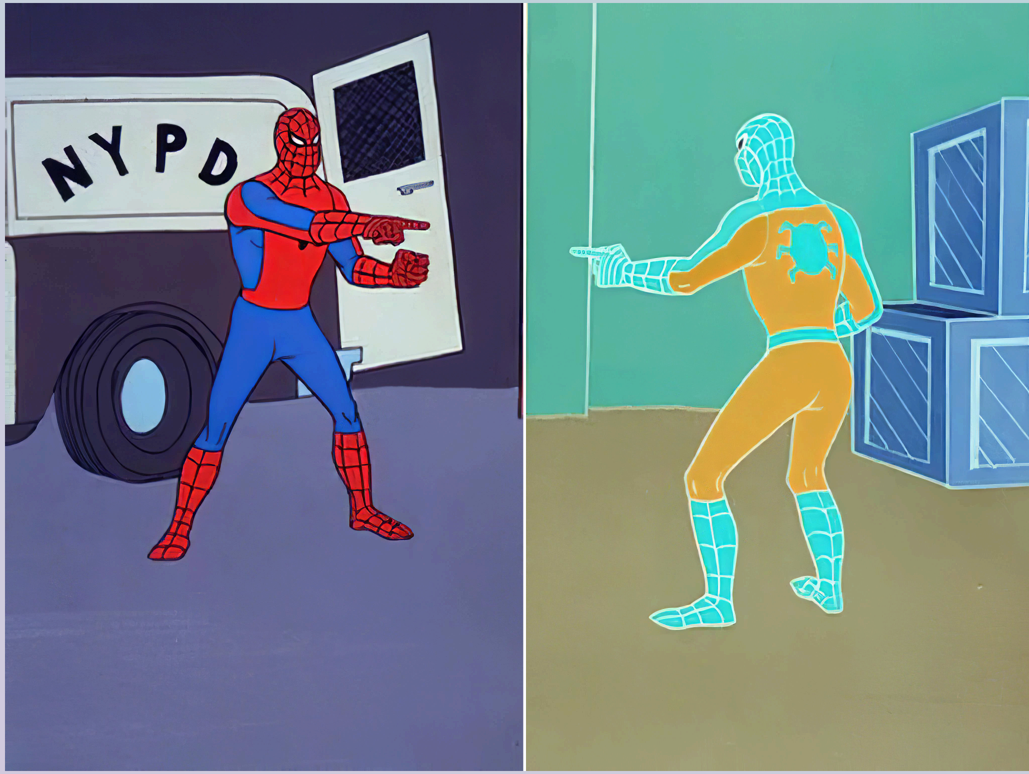
- If the high temperature of yesterday was 21 and today's high is 18, how much did the temperature change by?
- If tomorrow's high is 21, how much will the temperature change from today?

- Both are 3, just in different directions
- Numbers have a sign (+ or -) and a magnitude (the number without the sign)
  e.g. 4 and -4 have the same magnitude
- 4 is actually +4 but positive numbers are the default so we leave off the '+'
- We also call the magnitude the absolute value - it even has its own symbol: $|-4| = |4| = 4$

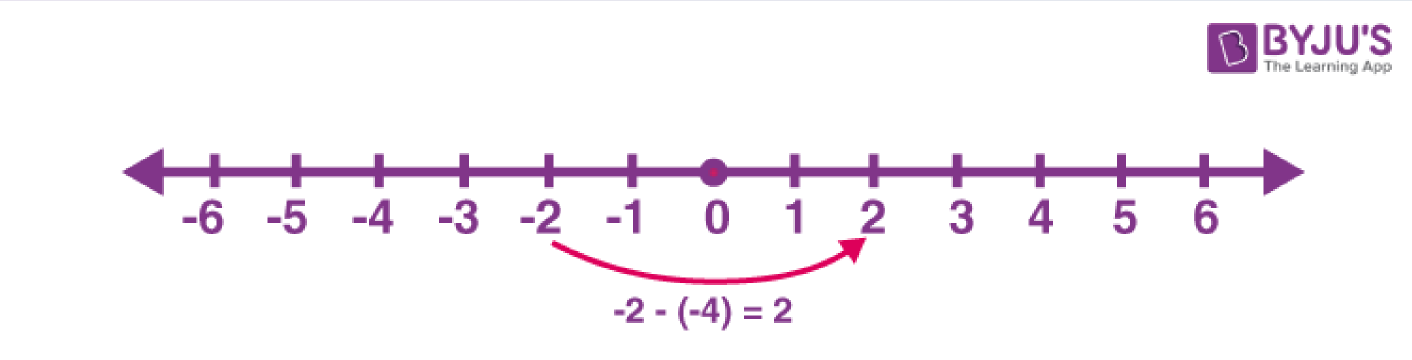


Negative numbers as Opposites

- Every positive number has a negative counterpart and vice versa
  e.g. 5 and -5 are opposites
- What's the opposite of 0.5?
- Does 0 have an opposite?



6B and 6C Adding or subtracting integers



We can use the number line to check which way we're going

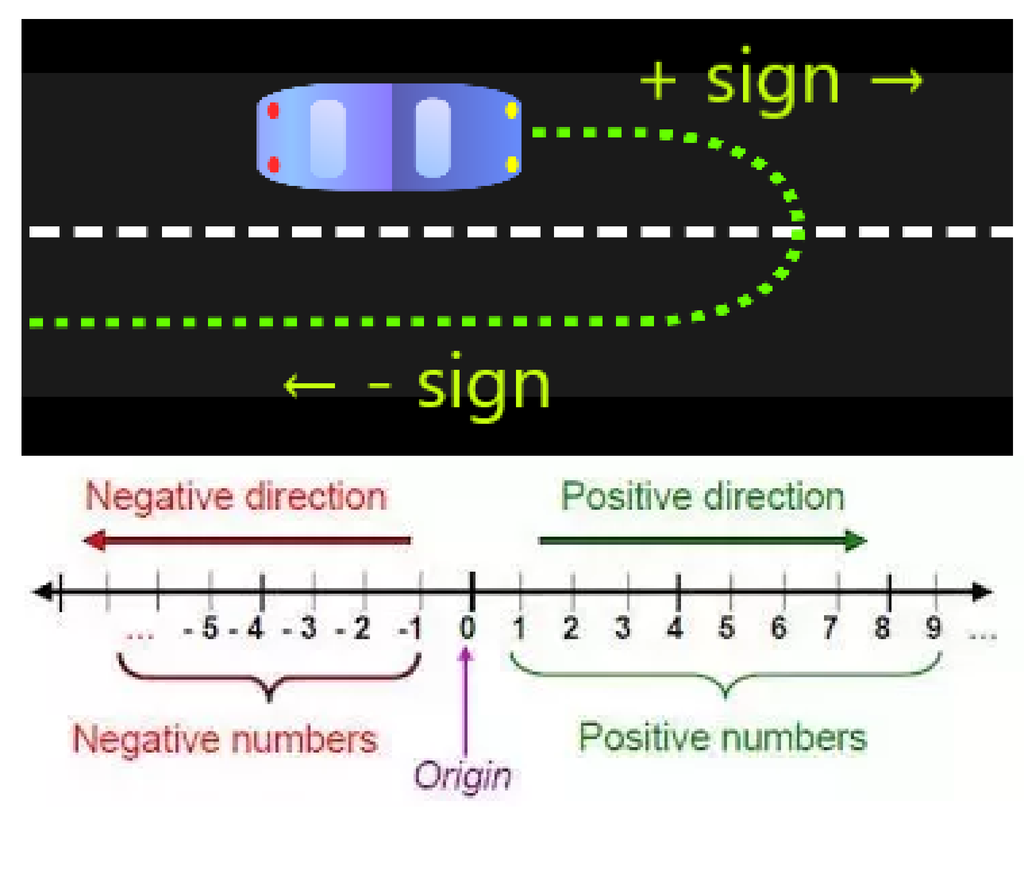
	Adding	**Subtracting**
Positive integer	Go right (bigger)	Go left (smaller)
Negative Integer	Go left (smaller)	Go right (bigger)

Combining signs

Alternatively: if there are two signs next to each other, we combine them

	Like signs	Different signs
	++	+-
	--	-+
Result	+	-

Sign conversions can be like driving: a negative sign is like a u-turn



Examples

First operand is positive

$5 + (+4) = 5 + 4 = +9$ (right of 5)

$5 - (+4) = 5 - 4 = +1$ (left of 5)

$5 + (-4) = 5 - 4 = +1$ (left of 5)

$5 - (-4) = 5 + 4 = +9$ (right of 5)

First operand is negative

$-5 + (+4) = -5 + 4 = -1$ (right of -5)

$-5 - (+4) = -5 - 4 = -9$ (left of -5)

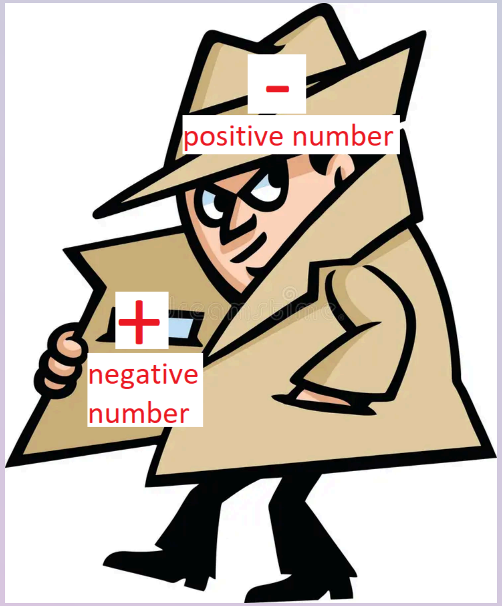
$-5 + (-4) = -5 - 4 = -9$ (left of -5)

$-5 - (-4) = -5 + 4 = -1$ (right of -5)

Subtraction: the secret Addition

What's the difference between 3p - 2 and 3p + (-2)?

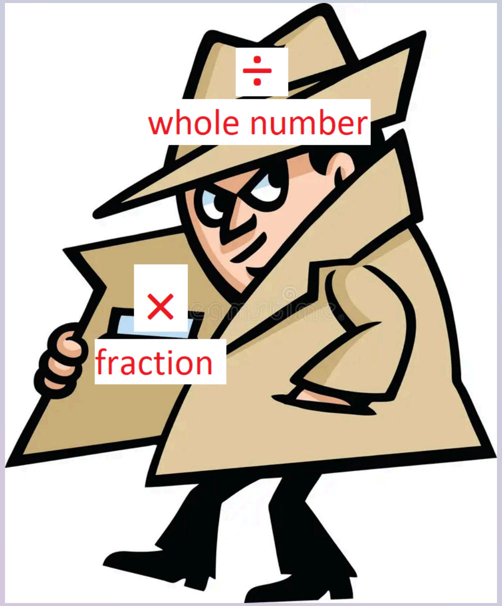
- If you said nothing, congrats, you're right!
- btw if this expression looks familiar to you, it was from your topic test on Algebra
-



Division

What about division? Can it be a secret operation too?

- Division: **the secret Multiplication**



- What's the difference between $a \div 8$ and $a \times \frac{1}{8}$?

but I digress: back to negative numbers

Examples (Your Turn)

Simplify

a) 39 - 42 =

b) 13 + 56 =

c) 36 - 3 =

d) 38 + 66 =

e) 52 - (-6) =

f) 29 - 91 =

g) 95 + (-11) =

h) (-48) - (-96) =

Answers

a. 39 - 42 = **-3**

b. 13 + 56 = **69**

c. 36 - 3 = **33**

d. 38 + 66 = **104**

e. 52 - (-6) = **58**

f. 29 - 91 = **-62**

g. 95 + (-11) = **84**

h. (-48) - (-96) = **48**

Recap

1. True / False: -8 > 5
2. Write in descending order: -9, 9, -2, 6, 0, 10
3. Evaluate each expression:
   a) 19) (-45) + 23 =
   b) 30 + 96 =
   c) (-16) + 20 =
   d) 85 + 33 =
   e) (-8) + 2 + (-10) =
4. Evaluate:
   a) 96 - (-49) =
   b) (-99) - (-26) =
   c) 41 - (-26) =
   d) 31 - 98 =
   e) 5 - 4 - (-1) =

6D Multiplying or dividing by an integer

Multiplying and combining signs

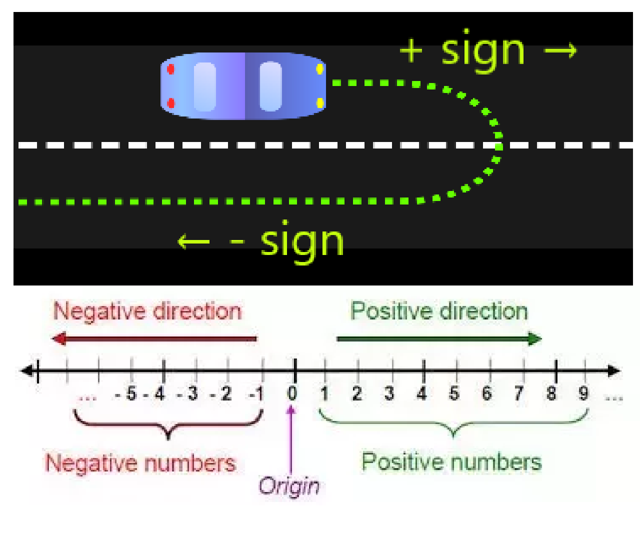
Just like we handled combining signs earlier

	Positive	Negative
Positive	*Positive*	Negative
Negative	Negative	*Positive*

So: *Like* signs: *positive*

Different signs: negative

Again, like driving: a negative sign is like a u-turn



Dividing

- Drop the signs and do the division like normal
- Then we determine the sign to put:
    - handle it similarly to multiplication

	Numerator *Positive*	Numerator Negative
Denominator *Positive*	*Positive*	Negative
Denominator Negative	Negative	*Positive*

Again, like multiplication: *Like* signs: *positive*

Different signs: negative

Example (We Do)

Which will be negative?

a. (-32) ÷ (-4) = b. 22 ÷ (-11) = c. (-24) ÷ 3 = d. 25 ÷ 5 =

Examples (You Try)

Evaluate:	
a. 9 x -8 =	b. 3 × -5 × -6 =
c. (-4) × -9 × -5 × 2 =	d. $-(9)^2$ =
e. 48 ÷ (-8) =	f. 6 x -4 × 5 =
g. $\frac{(-20)}{(-5)}$ =	h. (-20) ÷ 10 =
i. (-66) ÷ 66 =	j. $\frac{-9\times 7}{-3}$

Answers

a. 9 x -8 = **-72**	b. 3 × -5 × -6 = **90**
c. (-4) × -9 × -5 × 2 = **-360**	d. $-(9)^2$ = **-81**
e. 48 ÷ (-8) = **-6**	f. 6 x -4 × 5 = **120**
g. $\frac{(-20)}{(-5)}$ = **4**	h. (-20) ÷ 10 = **-2**
i. (-66) ÷ 66 = **-1**	j. $\frac{-9\times7}{-3}$ = **21**

6E Order of Operations

🤔 Why do we use brackets? When do we use brackets?

Brackets	May contain negative signs or need to be evaluated first
Index	Be careful of negative signs
Division & **M**ultiplication	Right to left
Addition & **S**ubtraction	Right to left after ÷ and ×

Examples (We Do)

a. $(-6)^3$ =

b. $(-5 \times 2 + 3)^2$ =

c. $-5^2 + 5$ =

Example 1 (Your Turn)

Evaluate:

a. $(-66)\div(-11)+(-9)+(-9)$ =

b. $(-3)-(-8)\times(-5)\times(-6)$ =

c. $(-10)\times(-7)\times(-4)+(-10)$ =

d. $[(-5)+8]\times(-2)-(-5)$ =

e. $10\times[90\div5-10]$ =

f. $(-12+9)^3$ =

Answers

a. $(-66)\div(-11) + (-9) + (-9)$ = **-12**
b. $(-3) - (-8)\times(-5)\times(-6)$ = **237**
c. $(-10)\times(-7)\times(-4) + (-10)$ = **-290**
d. $[(-5) + 8]\times(-2) - (-5)$ = **-1**
e. $10\times[90\div5 - 10]$ = **80**
f. $(-12 + 9)^3$ = **-27**

Examples 2 (Your Turn)

Evaluate:

a. $[(-2)^3 + (-7)] \times (-2) - (-3)$ =

b. $(-2) - [(-4) \div (-2)]^2 \times (-8)$ =

Answers

a. $[(-2)^3 + (-7)] \times (-2) - (-3)$ = **33**

b. $(-2) - [(-4) \div (-2)]^2 \times (-8)$ = **30**

Activity: Integer Race

Play in groups of 2 or 3. Each group has a number line and one counter per person.
Your **aim** is to get to **-47** *or* **+47** first.

How to play

- You start with a score of zero
- In each turn, you **roll** a die **twice** and get two numbers in order, e.g. say I roll 4 and then 6
- With those two numbers (in order), you use **two different operations** to change your current score, that is, add, subtract, multiply, divide. In my example, I could do: $0 + 4 \times 6$
  - Also: **at least one** of your two numbers must be **negative**, i.e. put a negative sign before one of them
    e.g. if I choose 6 to be negative, then I could have $0 + 4 \times -6$
  - You can insert **brackets wherever** you want
- After checking your calculation with your partner, place your counter at your new score.
- Now it's your partner's turn and they do the same thing

Integer Race, cont.

Draw a table in your notebook to keep your numbers straight while doing the operations

1. You should start with a current score in the first column, for each row
2. Write your rolled numbers down in the third and fifth columns
3. Choose your negative number and choose your operations
4. Calculate the result and write the result in the first column of the next row

Current Score	Operation 1	1st roll	Operation 2	2nd Roll
0	-	1	+	-1
[-2	-	1]	×	-6
18	+	6	-	-1
[25	-	1]	×	4
96	÷	[-1	-	1]
-48	-	-3	+	-2
-47	I win!			

6F Substitution

Back to algebra we go!

Do you remember:

> **Substitute**: replace the pronumerals with a number and then calculate the result

- Now with negative numbers!
- Tip: if substituting a pronumeral with a negative number, use brackets
  - This will avoid confusion about combining signs

Examples (We Do)

4s + 3f + 6	when s = -2 and f = -3
$5s - \frac{h}{-5}$	when h = -15 and s = -4
8(-4n - 2h) - 9	when h = 9 and n = -7

Examples (Your Turn)

Evaluate each expression for a = 2, b = −5 and c = 3

a. a + 3	b. −3c + 5	c. 2c − 4
d. $ab - c^2$	e. (a – b)(b – c)	f. $\frac{c-b}{a}$
g. $2b^3$	h. $(2b)^3$	

❤️ ANSWERS ARE MIXED UP, check them

Answers

Evaluate each expression for a = 2, b = −5 and c = 3

a. a + 3	b.= **-13**	b. −3c + 5 = **−4**
d. $ab - c^2$ = **19**	e. (a – b)(b – c) = **-56**	f. $\frac{c-b}{a}$ = **4**
g. $2b^3$ = **-250**	h. $(2b)^3$ = **-1000**	

Challenge

When converting from Fahrenheit to Celsius, we use the formula

$$C = \frac{5}{9}(F - 32)$$

Find the one temperature that is the same numerical value in both Celsius and Fahrenheit.

- Hint: Set C and F to both be x

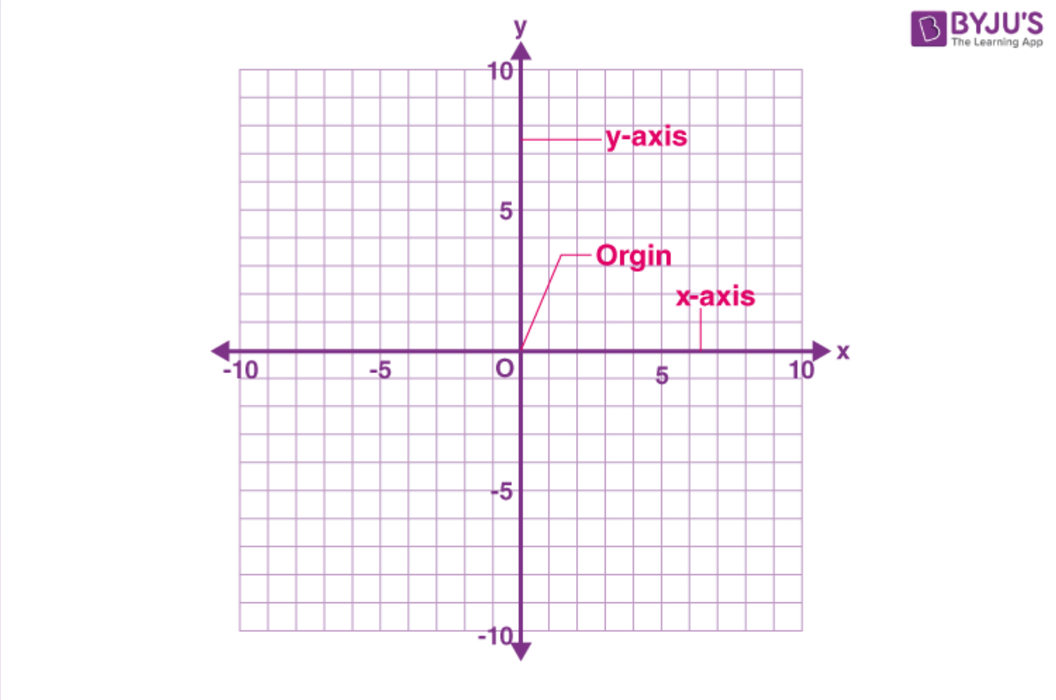
Extension

1. Factor $\frac{-x}{x}$
2. Factor $\frac{h-2}{2-h}$
3. Factor $\frac{x^2-x}{1-x}$

6G Cartesian Plane

If a number line is so great, why don't we have two of them?

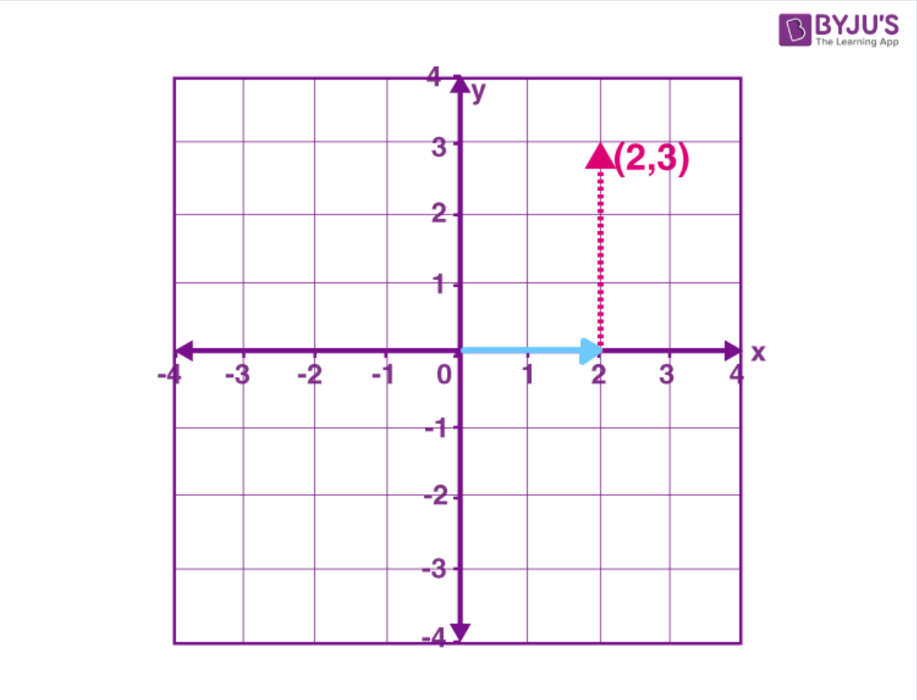
- Now we do.
- Introducing: The Cartesian Plane (also called the number plane)



The horizontal number line is called the x-axis and vertical number line is called the y-axis
They meet at the origin

Coordinates

We can find points by a pair of numbers, called coordinates

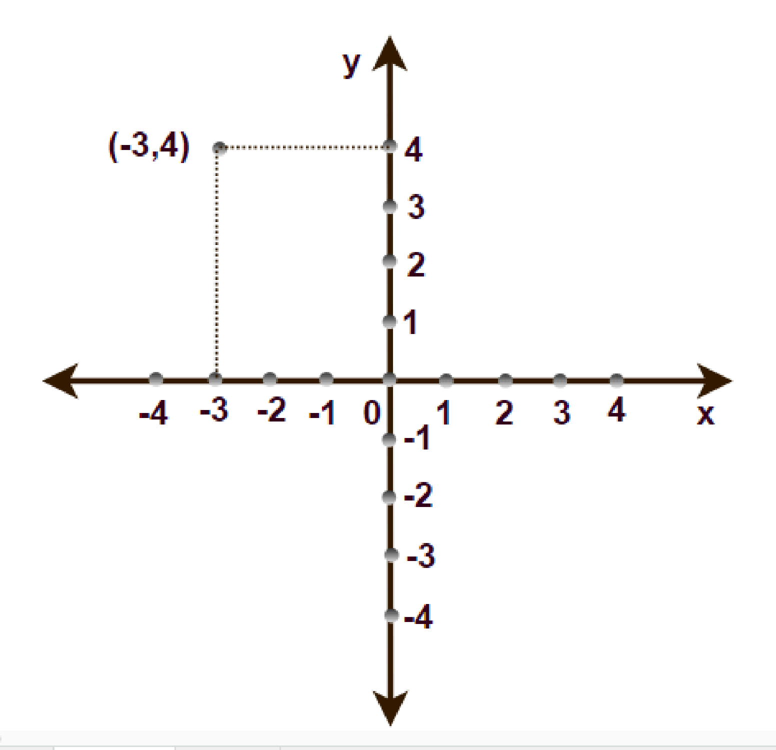
- e.g. (2,3)
- We go along the x-axis: 2 points right, and then up the y-axis
- 

- 🤔 What are the origin's coordinates?
- We always start at the origin for plotting points (🤔 Why?)

Negative Coordinates

What if one of our coordinates is a negative number?

- e.g. (-3, 4)
- We will still go along the x-axis, but to the **left** instead
- Do we still go **up** the y-axis?
-



Negative Coordinates (more)

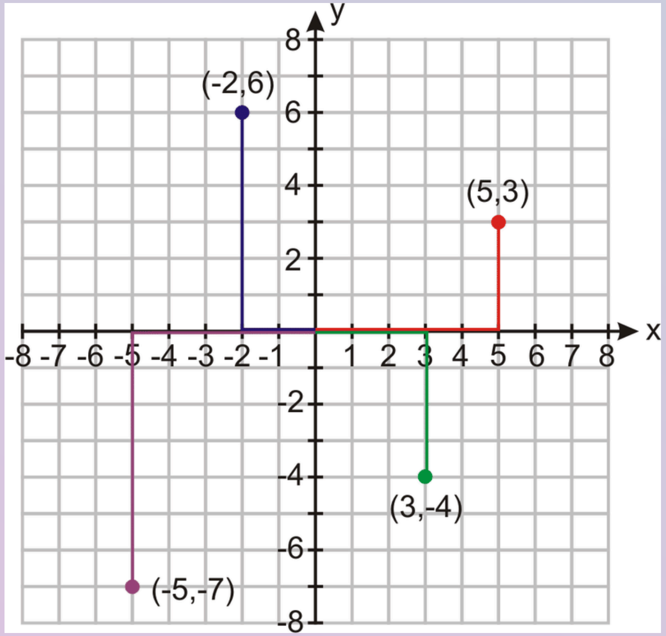
How about when we have negative y-coordinates?

- We go down instead of up

What about when both coordinates are negative?

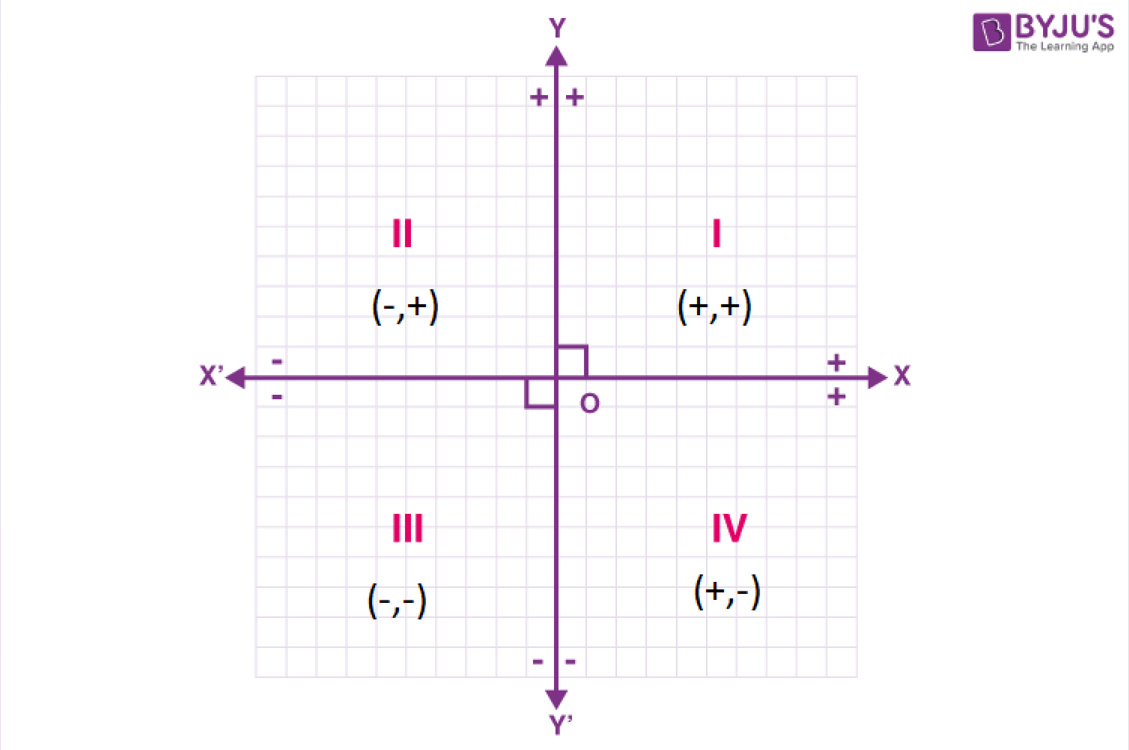
- We go left along the x-axis and down the y-axis

Examples of all four cases:

- 

Quadrants

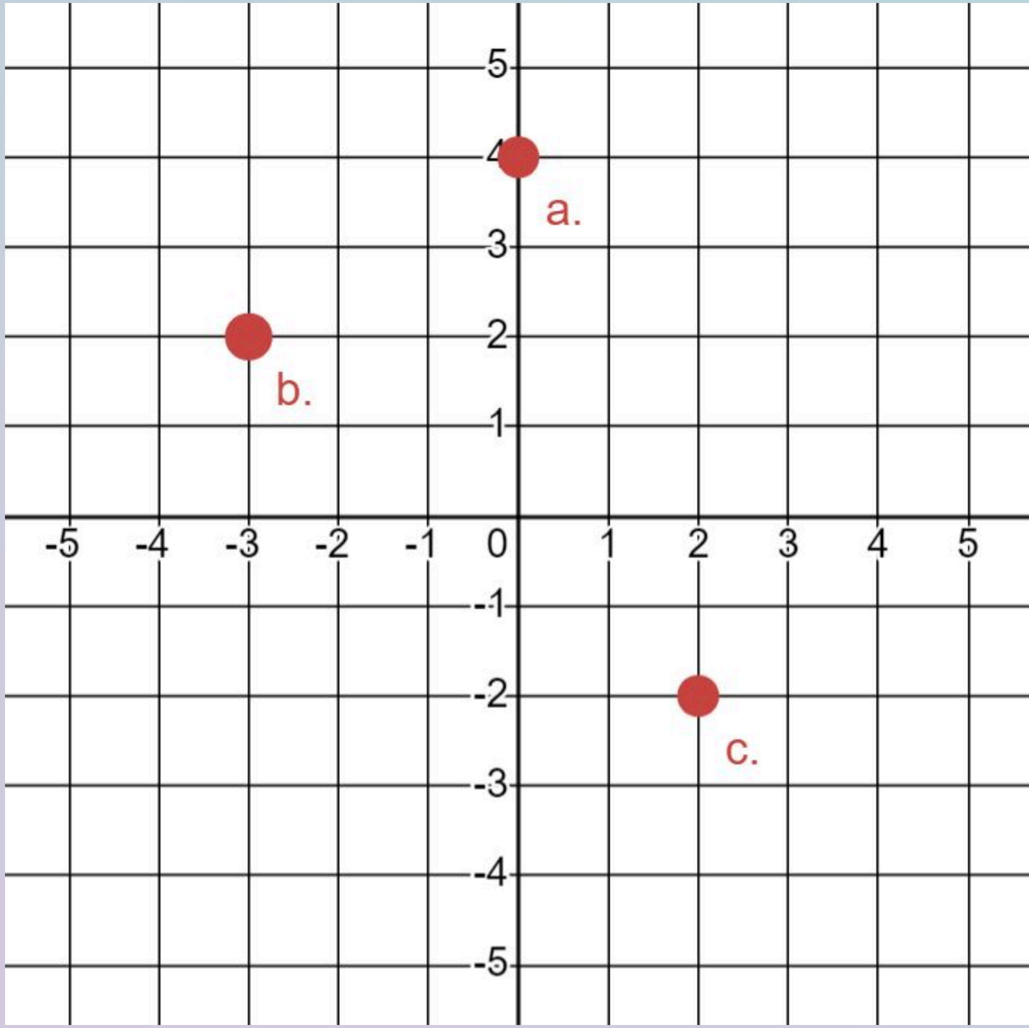
We divide the Cartesian plane into four parts, based on the sign of the x and y-coordinates



We number the quadrants in an anti-clockwise direction

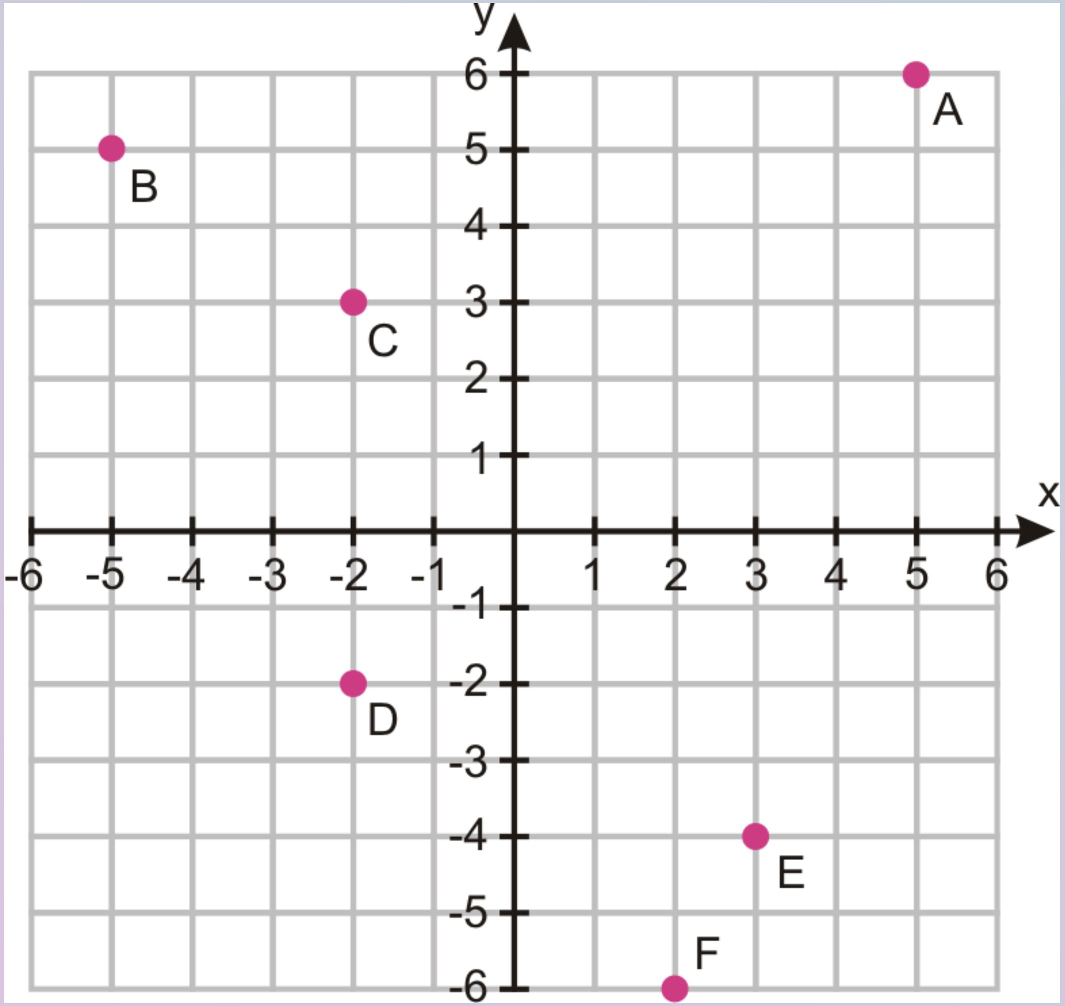
Identifying Points

- First, find the point straight above/below the point on x-axis: this is our x-coordinate
  - e.g. for b., we go down to -3
- Then find the point straight to the left/right on the y-axis: this is our y-coordinate
  - e.g. for b., we go right to 2
- Put the numbers in the format (x,y) and that's our answer
  - So, b.'s coordinates are (-3,2)



Examples (Your Turn)

Identify the following points:



Answers

A: **(5,6)**

B: **(-5,5)**

C: **(-2,3)**

D: **(-2,-2)**

E: **(3,-4)**

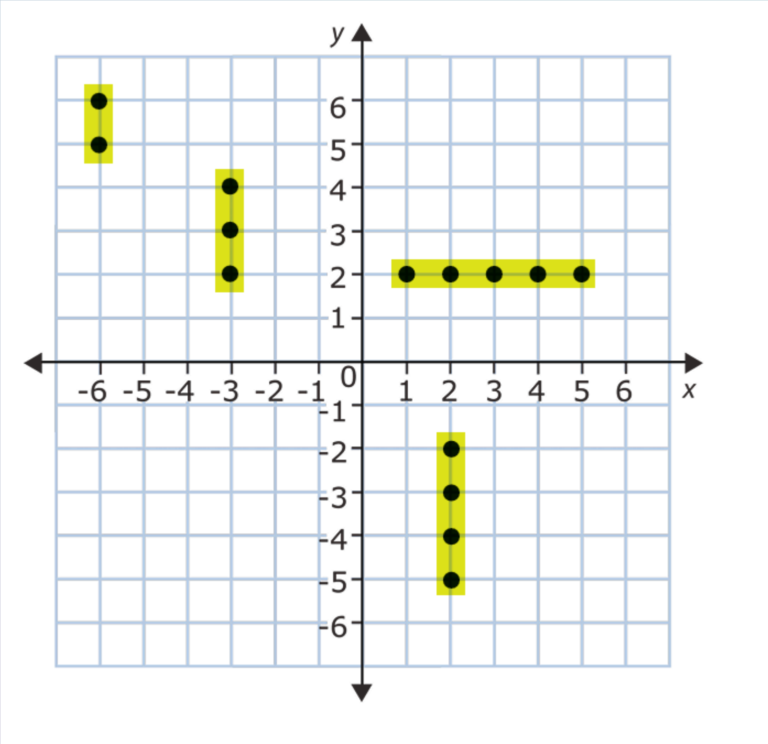
F: **(2,-6)**

Battleship

1. To set up, mark the locations of your ships on the plane called "Your Ships and Your Opponent's Shots" with highlighter, hiding the board from your partner. Ships can be diagonal, vertical, or horizontal. You each have 4 ships, each with different numbers of points on the coordinate plane.
2. Player 1 begins guessing where the other student has placed their points: call out a single ordered pair.
3. Player 2 calls out "hit" or "miss"

- If Player 1 makes a hit, they put an X on that point on the plane called "Your Shots". Player 2 puts an X on their plane called "Your Ships and Your Opponent's Shots".
- If Player 1 missed, they put a dot on the specified coordinate on their top graph. Player 2 does not do anything in this case.
- Use one coloured pencil for hits and the other for misses (or different symbols if you don't have different coloured pens).

5. If a player has hit the last point on the ship, the opponent says: "You sunk my ship!"
   The first player to sink all their opponent’s ships wins!



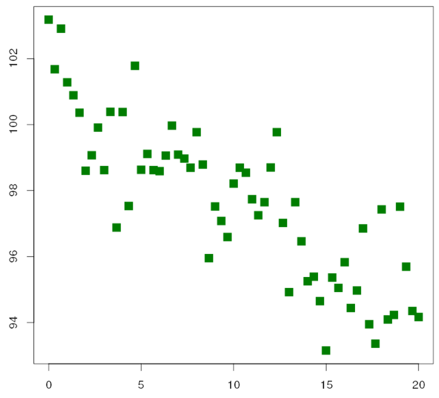
Cartesian Plane: More than dots

Earlier we located and identified points in the Cartesian Plane

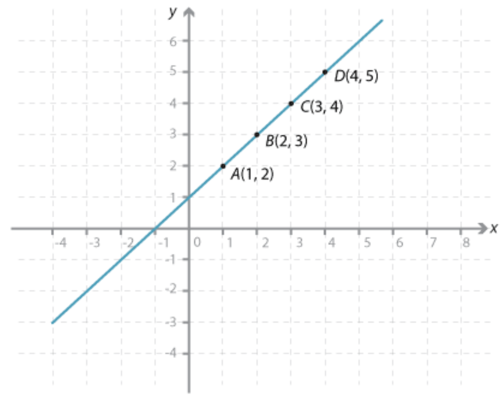
We can also draw lines in the Cartesian Plane!

- Remember: a line is just many, many points, but they can't just be anywhere - they have to be lined up

  How do we tell if they are a line?



Not a line

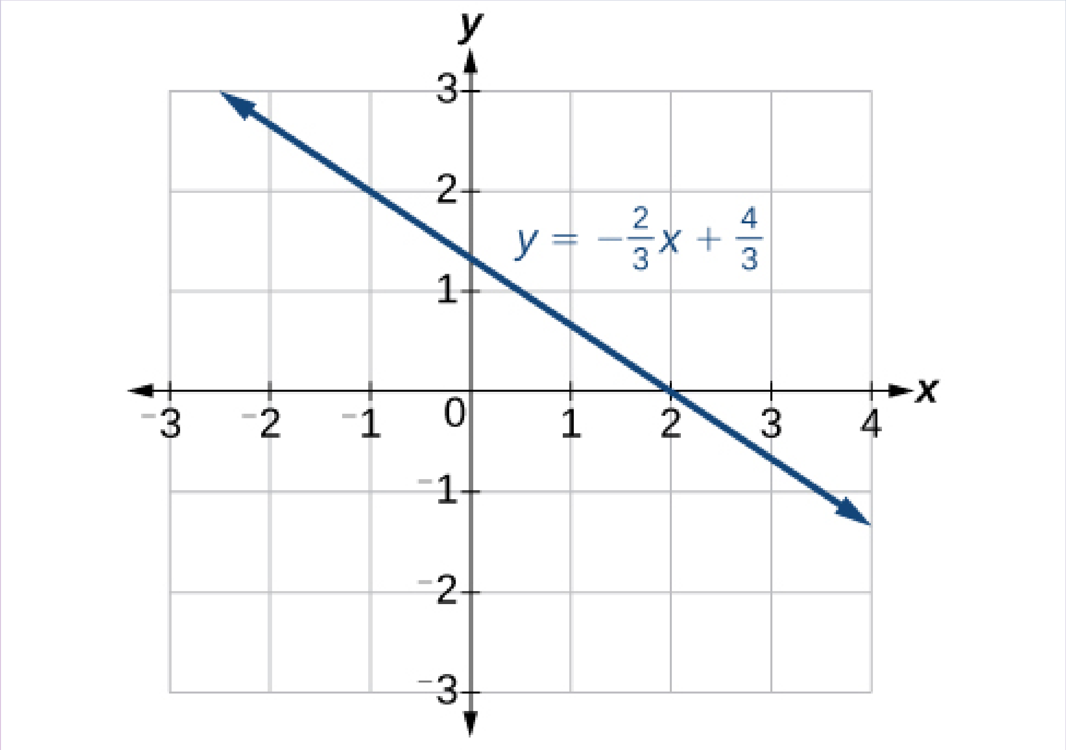


A line

Lines and Equations

Lines in the Cartestian plane can be represented as an equation between x and y, such as y = 2.5x

- 🤔: Why?



Checking if a point is on a line

Is the point (1, 3) on my line $y = 2.5x$? How do we find out?

We substitute the values in: $x = 1$ and $y = 3$ for our dot

On my line, when $x = 1$
$y = 2.5{\times}1$

= 2.5

2.5 $\neq$ 3

So (1,3) is not on the line!

Examples

Check if the following points are on the lines listed

Point	Line	Point	Line
a. (3,7)	$y = 2x + 1$	b. (-1,0)	$y = 2x + 1$
c. (1,2)	$y = -x + 3$	d. (1, 3)	$y = 5x - 2$
e. (1, 1)	$y = 5x - 5$	f. (2, 8)	$y = 5x - 2$
g. (-1, -10)	$y = 5x - 5$	h. (2, 5)	$y = 1.5x$

Relationships

- We say two variables are related when changing the value of one affects the other.
- Wait, what's a variable?
    - A pronumeral can represent a quantity that can change, e.g. how many mangoes I'm buying today. When we use a pronumeral in this way, we call it a variable. For example, x could be the number of mangoes I buy.
- When two variables are related, we can express one in terms of the other, that is
    - I can say $y = 2.5x$ for a situation where y is the cost of my groceries and I'm buying x mangoes for $2.5 dollars each.

Linear Relationships

- The simplest kind of relationship is a linear relationship
- **Linear relationship**: a relationship between two variables such that they both have an index of 1 in the equation expressing one in terms of the other
    - More concretely, we can say y and x have a linear relationship if we can write $y = mx + c$ for some constant numbers m and c
    - In the example I used ($y = 2.5x$), $m = 2.5$ and $c = 0$

Tying it back to Lines

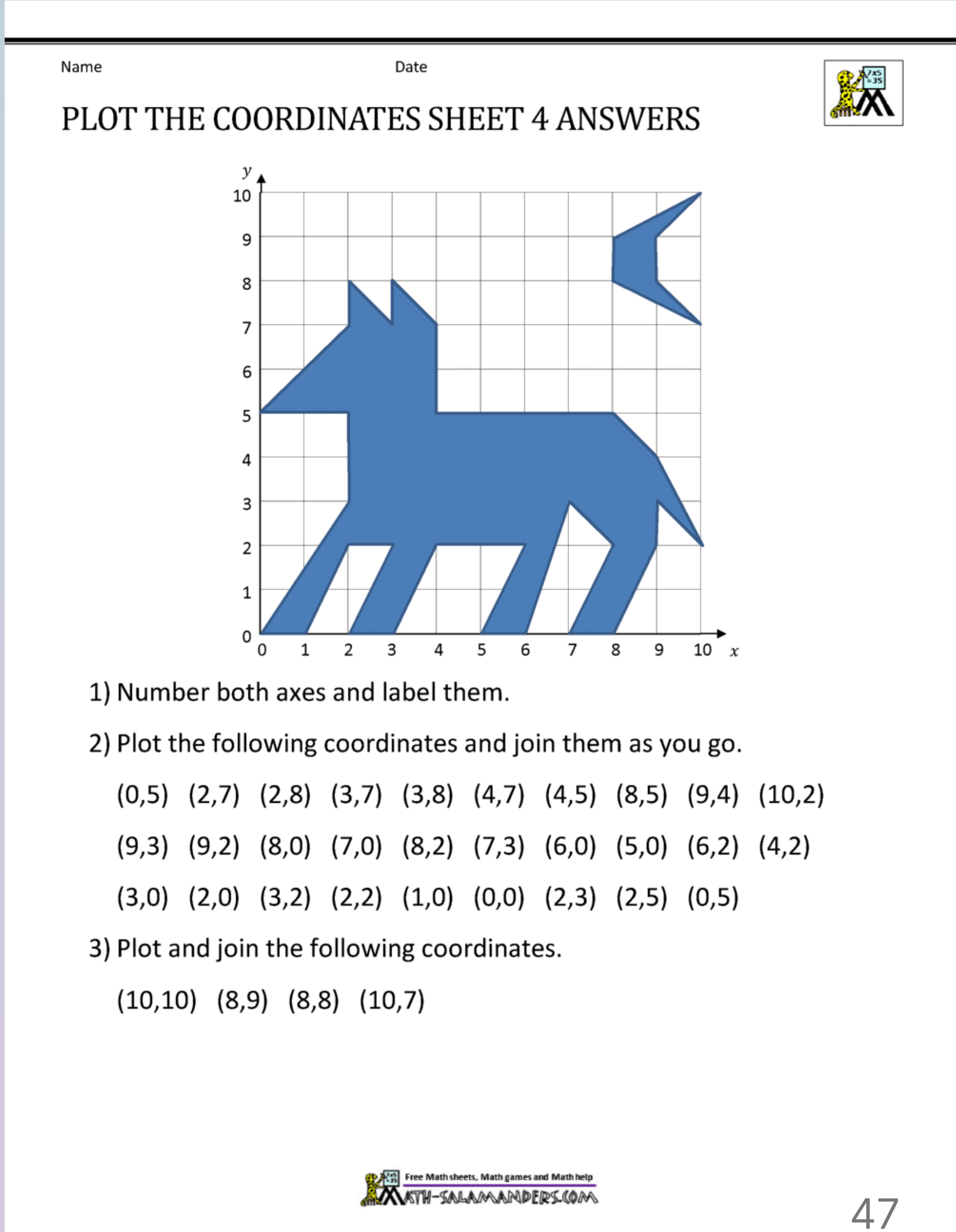
- If that equation looks familiar to you, it's the one we looked at when drawing lines in the Cartesian plane
    - *Line*ar relationships are represented by a *line* when we draw them
- different representations of the same concept:
    - Equations in x and y
    - linear relationships
    - lines in the Cartesian Plane

Transformations in the Cartesian Plane

Now we've done things like plot points and draw lines in the Cartesian plane

We can connect dots to make shapes and even images

And when we make shapes, we can transform them!


Name Date

PLOT THE COORDINATES SHEET 4 ANSWERS

1) Number both axes and label them.

2) Plot the following coordinates and join them as you go.

(0,5) (2,7) (2,8) (3,7) (3,8) (4,7) (4,5) (8,5) (9,4) (10,2)

(9,3) (9,2) (8,0) (7,0) (8,2) (7,3) (6,0) (5,0) (6,2) (4,2)

(3,0) (2,0) (3,2) (2,2) (1,0) (0,0) (2,3) (2,5) (0,5)

3) Plot and join the following coordinates.

(10,10) (8,9) (8,8) (10,7)

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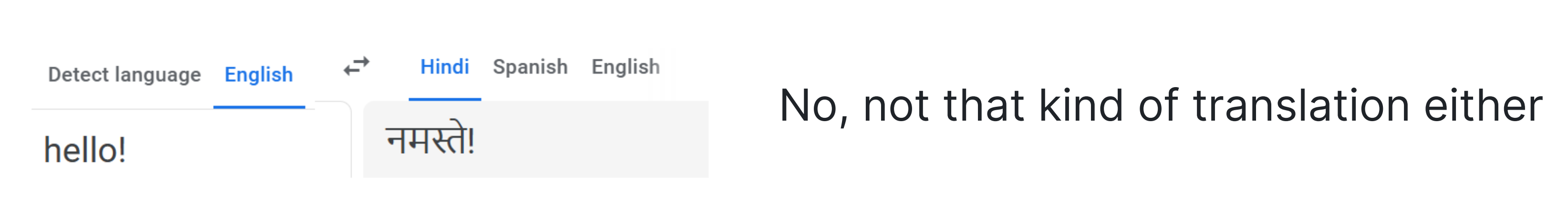
No, not that kind of transformation

A **transformation** in geometry means changing a shape's **position** or its **direction** in the Cartesian plane

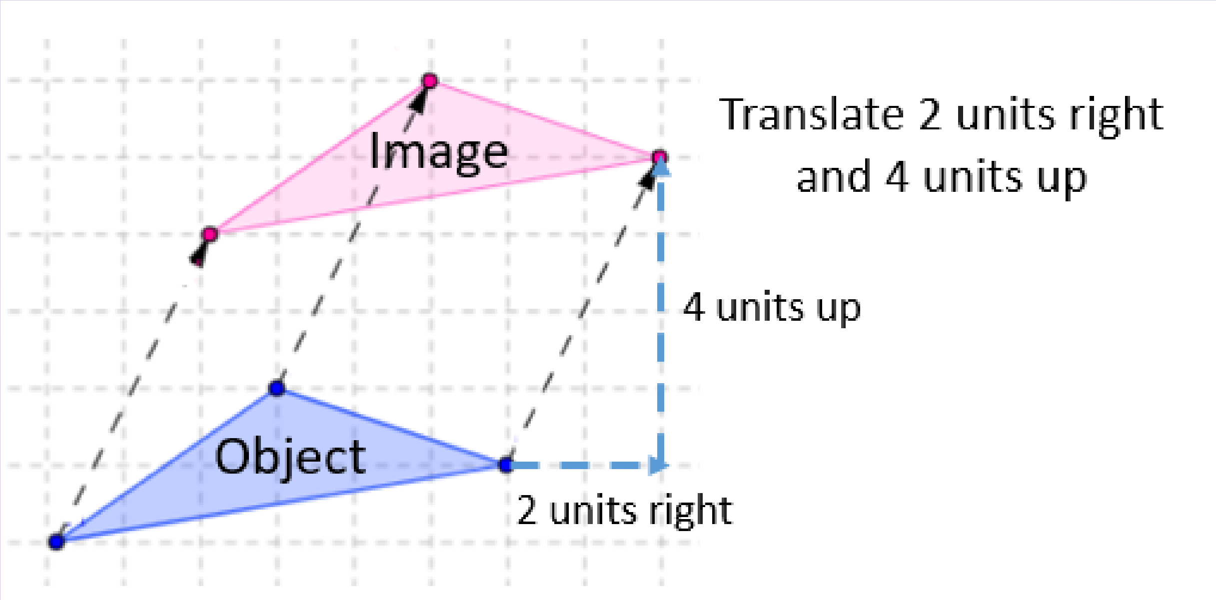
There are four kinds:

1. Translation
2. Rotation
3. Reflection
4. Dilation

Translation



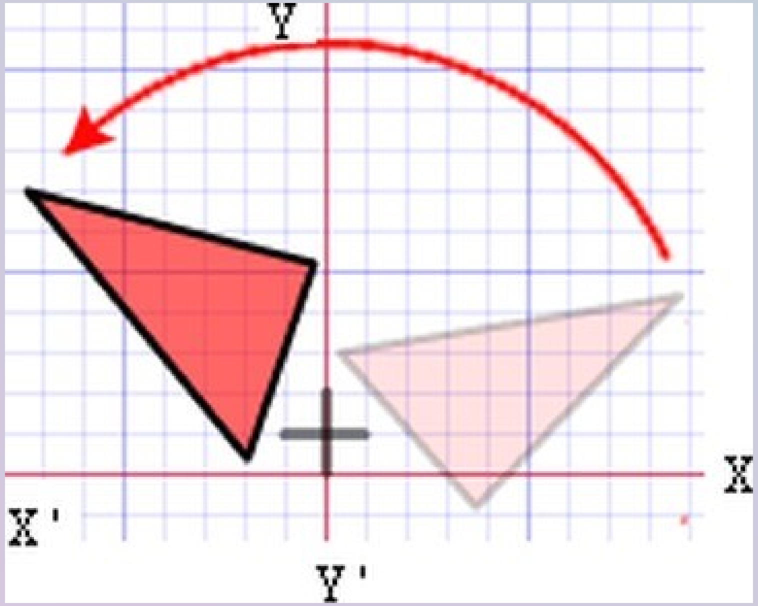
Translating a shape means moving it, whether left, right, up, down or a combination of two directions.



Each point of the shape has to be moved the same distance in the same direction

Rotation

Rotating a shape means turning it around a fixed point (which we call the centre of rotation)

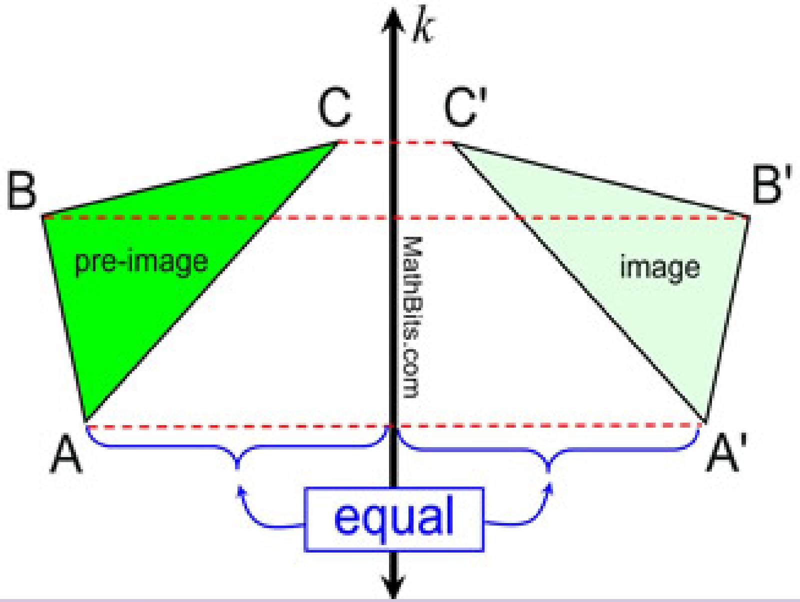


- 🤔: How many ways can we rotate an object?
    - 🤔: How much can we rotate an object?
    - 🤔: How many centres of rotation can a shape have?

Reflection

Reflection means creating a mirror image of a shape across a line

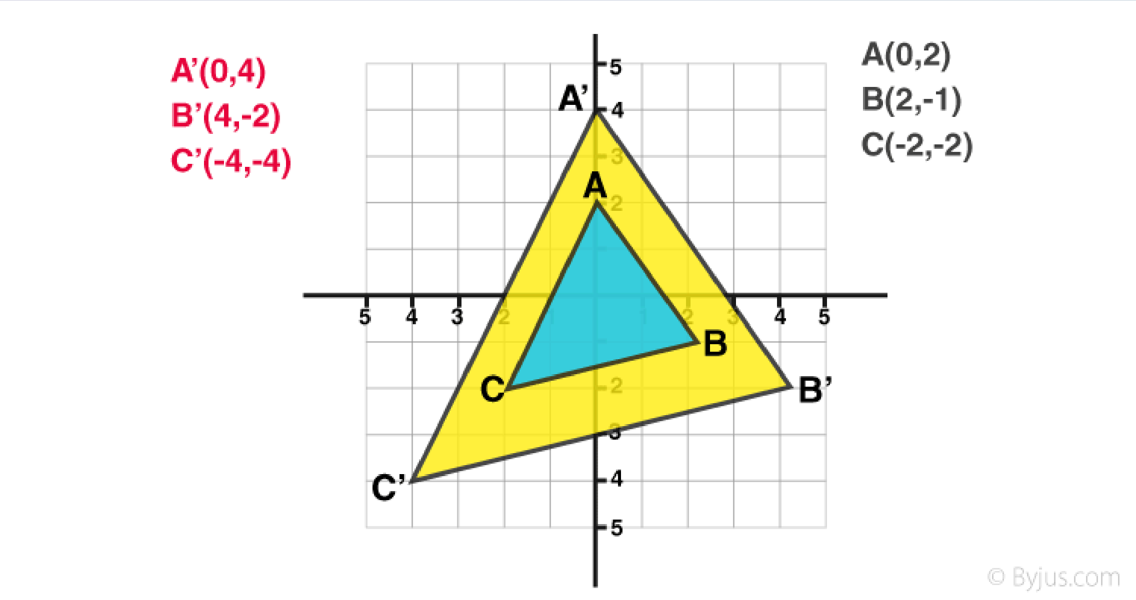
- Each reflected point is the same distance from the line as the original point
- But the overall direction of the shape is swapped



- 🤔: Do we have to reflect across the y-axis?
- 🤔: What does it look like if we reflect a square across a parallel line?

Dilation

Dilation means resizing a shape without changing it otherwise

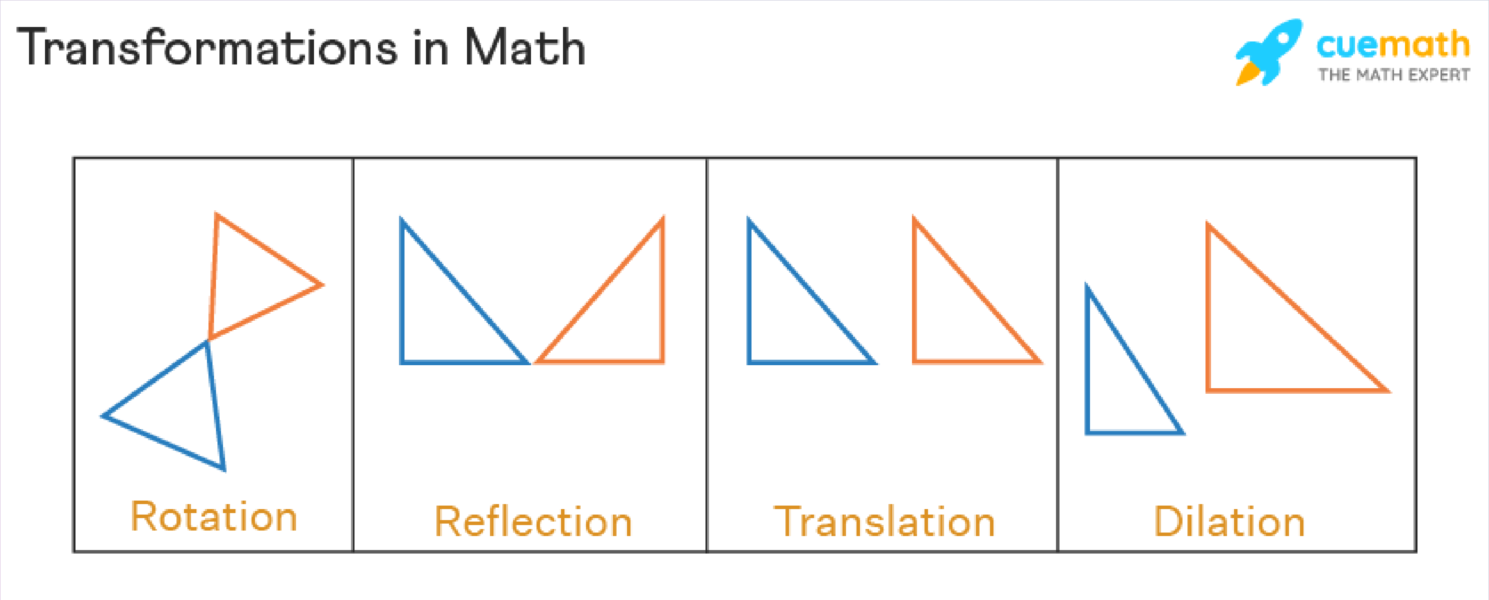


All lengths must be changed by the same multiplier (called the **scaling factor**)

- 🤔: Why are lengths multiplied by the scaling factor instead of added to?
- 🤔: Do the angles change?
- 🤔: How is dilation different from the other transformations?

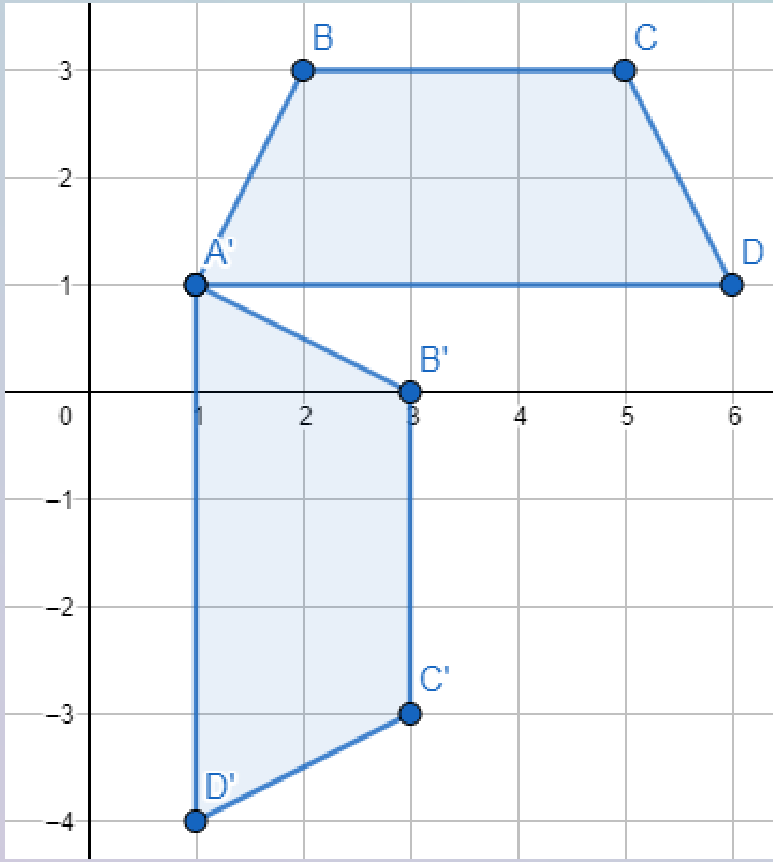
Transformations: Summary

- We often use the term **object** or **pre-image** for the shape before transformation
- And after transformation, we call the result the **image**



Identifying the transformation

1. Is the new shape at the same angle?
2. Is it the same size?
3. Is it facing the same direction?
4. Is it in the same location?



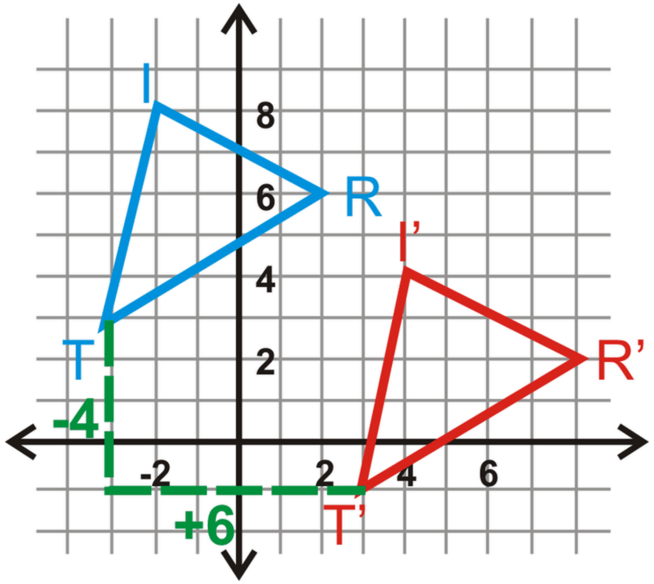
Applying a Transformation

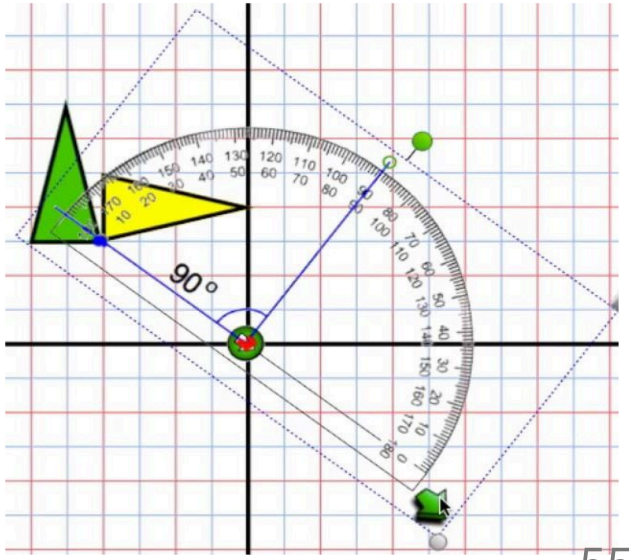
Translation

- Add how many units to the right to the x-coordinate and how many units up to the y-coordinate, to the respective coordinates for each corner

Rotation

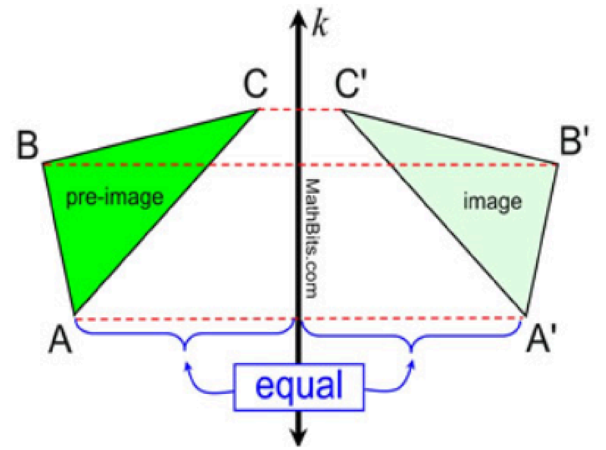
- For each corner: measure the distance from the centre and then place the given angle of rotation, from the centre, between the old corner and the new corner





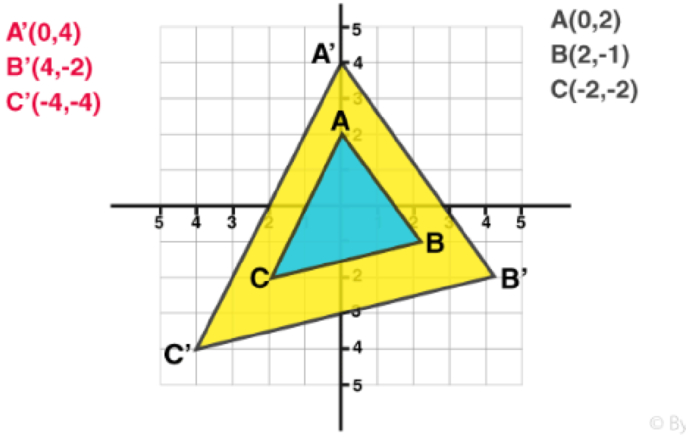
Reflection

- Measure each corner from the axis of reflection and then place out the new points at the same distance in the opposite direction



Dilation

- Multiply both the coordinates of each corner with the scaling factor, to find the coordinates of new points



Thank you for a wonderful term together!