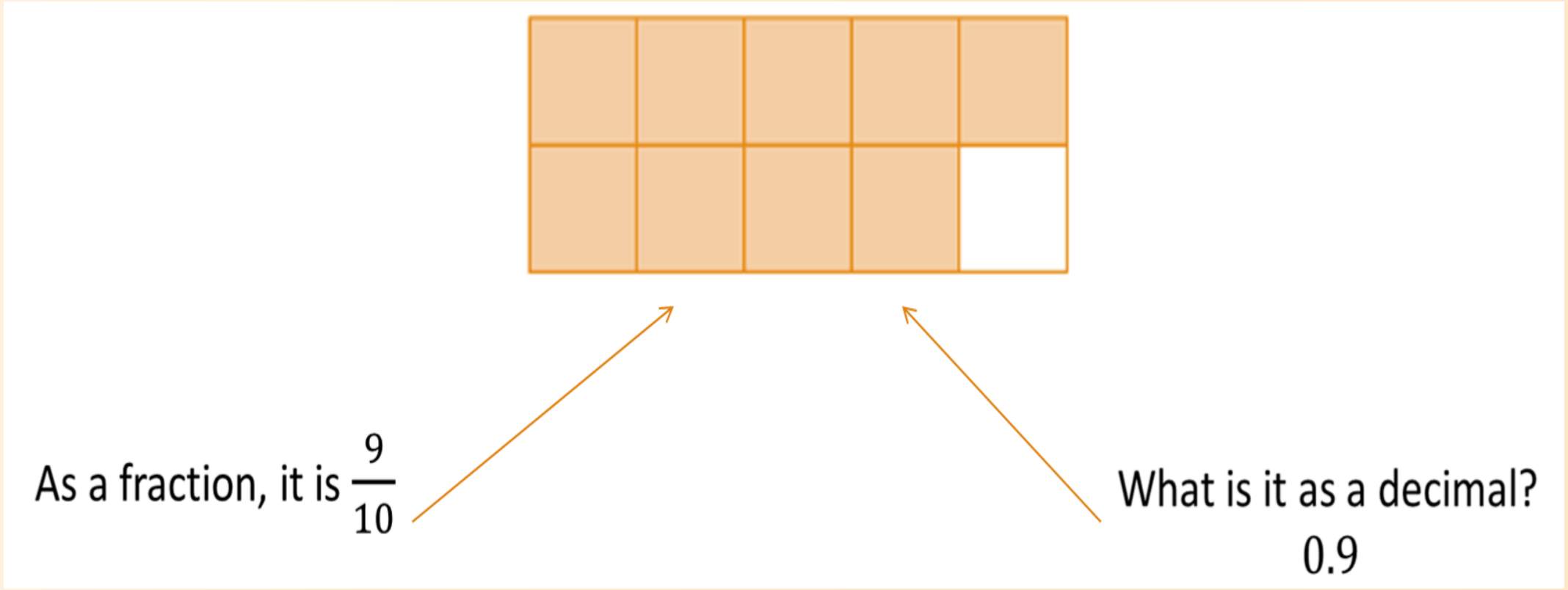
Welcome to Decimals!

Decimus meant tenth in Latin

What is a decimal?

- A decimal is a special type of fraction where the denominator is a power of ten such as 10, 100 or 1000.
- 

Is a decimal a rational number?

Let's remember: What's a rational number?

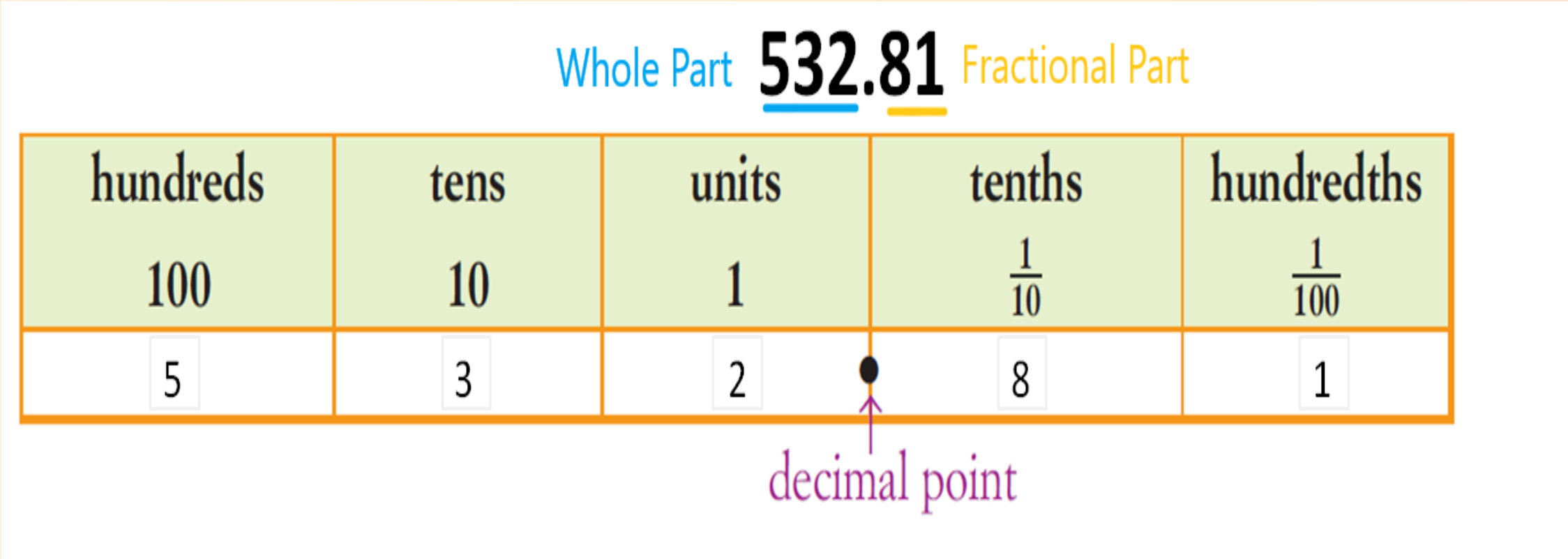
- A number that can be written as $\frac{a}{b}$ where a and b are integers and $b \neq 0$

So, is a decimal a rational number?

- Yes!
- Unless it never terminates or repeats, a decimal is a rational number
- A decimal like 0.42569 can be written as
  - $\frac{42569}{100000}$

Place Value

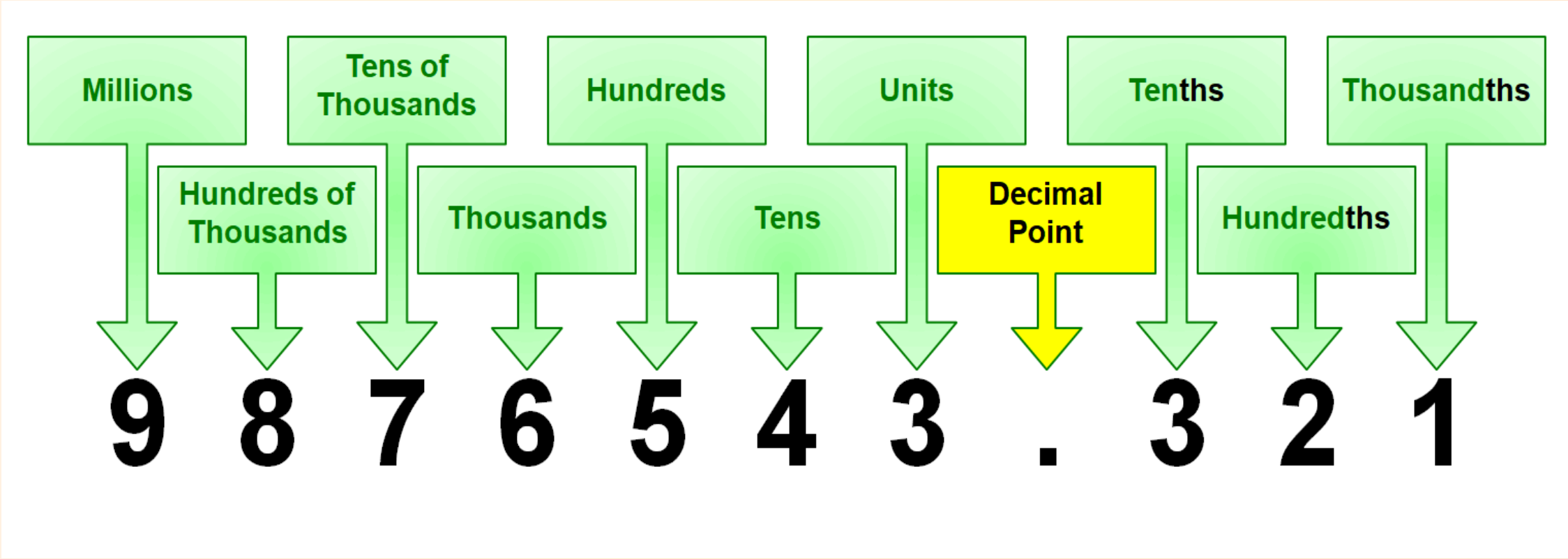
The position of a digit in a number shows its size. This is called the place value.



The position of a digit in a number gives the value of that digit.

The further **left** the digit the **greater** its value.

- Let's read out this number together



Examples (You Try)

Write the value of the bolded digit: answer using both numbers and words

🟢	a) 5**7**30.92	b) 9**1**843.48	c) 740**8**3.96
	d) 8521.**6**3	e) **6**1293.475	f) 70045.29**3**
🔴	g) **2**636897.831	h) 5**7**166.02	i) 4**2**35.8
	j) 295.3**8**65	k) **4**8738.2	l) 82747.1
⭐	m) 1.4893**5**	n) 234.2355**8**	o) 32453.24457

⬆️ Use only numbers for the last three questions ⬆️

Answers

🟢	a) 700	b) 1000	c) 80
	Seven hundred	One thousand	Eighty
	d) 0.6	e) 60000	f) 0.003
	Six tenths	Sixty thousand	Three thousandths
🔴	g) 2000000	h) 7000	i) 200
	Two million	Seven thousand	Two hundred
	j) 0.08	k) 40000	l) 7
	Eight hundredths	Forty thousand	Seven
⭐	m) 0.0003	n) 0.0005	o) 0.00007

Ordering decimals

- Easy mode: if we have the same number of digits after the decimal point.
- Harder mode: any number of digits
  (old-style computer sorting of files gets confused about this)
- So, we'll make all our numbers have the same number of digits
- Example 1: put these decimals in order of size from highest to lowest.

0.12 0.302 0.5 0.06

Step 1	Look at the decimals to find which one has the most number of digits after the decimal point.	0.12	0.302	0.5	0.06
Step 2	Add zeros to the other decimals so that they all have this number of digits after the decimal point.	0.120	0.302	0.500	0.060
Step 3	Put the numbers in order of size.	0.500	0.302	0.120	0.060

Example 2 (We Do)

Put the decimals in Ascending Order:

0.903	0.2	0.54	0.37

Step 1	**Look at the decimals to find which one has the most number of digits after the decimal point.**
Step 2	**Add zeros to the other decimals so that they all have this number of digits after the decimal point.**
Step 3	**Put the numbers in order of size.**

Questions (You Do)

Put these decimals in order from lowest to highest:

a) 4.6 | 4.06 | 4.26 | 4.16

b) 7.28 | 7.8 | 7.08 | 7.18

c) 5.06 | 5.6 | 5.62 | 5.26

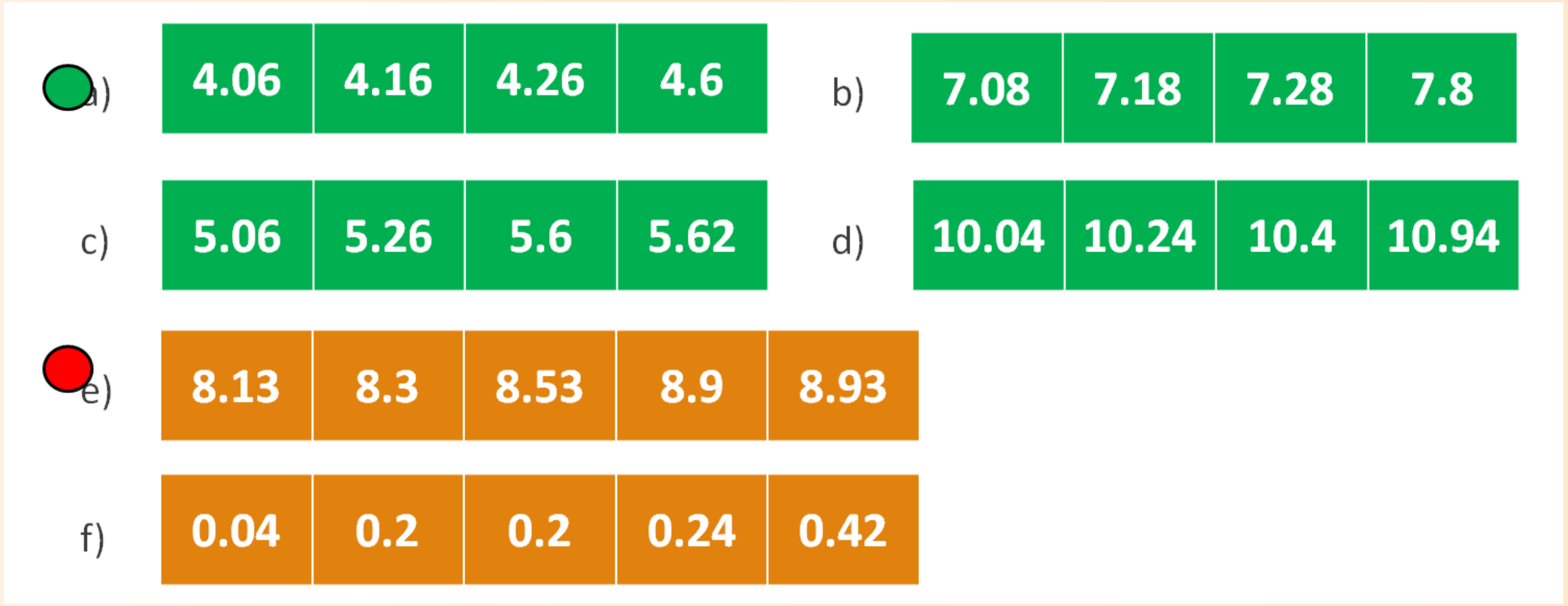
d) 10.94 | 10.24 | 10.04 | 10.4

e) 8.53 | 8.13 | 8.93 | 8.3 | 8.9

f) 0.42 | 0.2 | 0.24 | 0.2 | 0.04

- Challenge: Use the numbers 1, 2 and 3 to make six decimals numbers. Then order them from smallest to largest. E.g. 1.32, 2.31

Answers


a)
4.06
4.16
4.26
4.6
b)
7.08
7.18
7.28
7.8
c)
5.06
5.26
5.6
5.62
d)
10.04
10.24
10.4
10.94
e)
8.13
8.3
8.53
8.9
8.93
f)
0.04
0.2
0.2
0.24
0.42

Questions

Put these decimals in order from highest to lowest:

a)	6.4	6.04	6.41	61.4	6.14
b)	3.27	32.7	3.07	3.72	27.3
c)	16.04	1.64	10.6	16.4	1.46
d)	42.32	42.04	42.3	42.23	40.32

- Challenge:

  Can you order following measurements in ascending order?

124cm	0.75m	65mm	1.4m	1.1km

Answers

a)	61.4	6.41	6.4	6.14	6.04
b)	32.7	27.3	3.72	3.27	3.07
c)	16.4	16.04	10.6	1.64	1.46
d)	42.32	42.3	42.23	42.04	40.32

- Challenge:

65mm < 0.75m < 124cm < 1.4m < 1.1km

Activity: Decimal Spread

- You each have 3 cards and 1 sheet of paper with place value slots
- You can make different decimal numbers by ordering the cards in the slots
- Now let's play a partner/group game based on the decimals you can make
- There are a few different ways to play:
  - Sorting: Play in groups of 3-4
    Each makes a number at random and then you all sort them in ascending order
  - Spreads: Play in pairs
    - Spread 1: Work with your partner to try to make your numbers as close together as possible
    - Spread 2: Try to make your numbers as far apart as possible
- After you've played your game of choice, shuffle the cards on your table and share them out again!
- If you want a harder mode, 4 card spread is also possible - flip your sheet and ask for more cards.

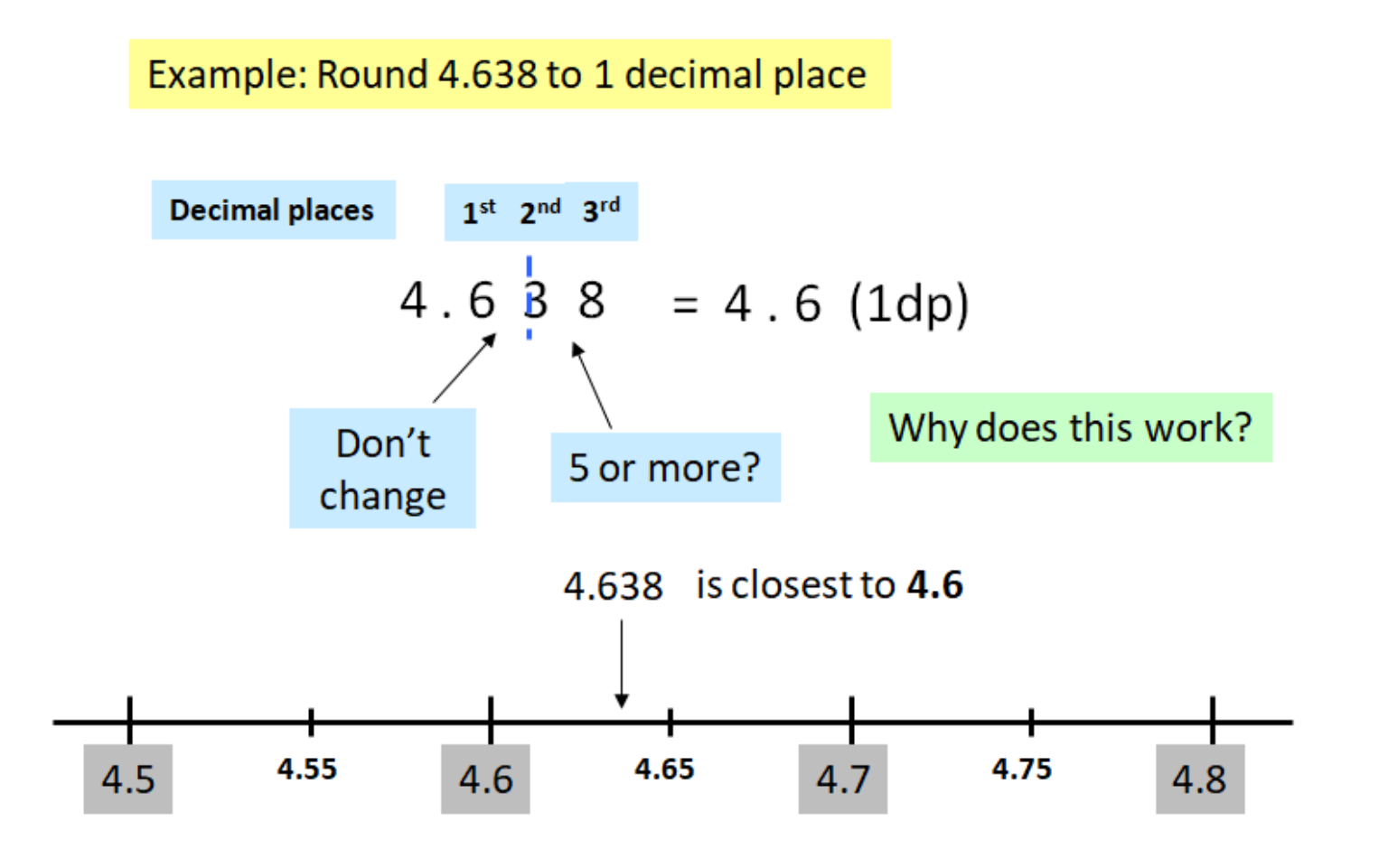
Leaving Activity: Ordering Yourselves

- Pick 4 digits from 1 to 10
- Make a number from them, no repeating digits
- Put a decimal point somewhere in the number
- Write it down on the sticky note I've handed out
- When everyone is done, we'll order ourselves by value of our numbers

5B: Rounding Decimals

🎵 4 or less, let it rest

5 or more, up the score 🎶



Why Do We Round?

- Money only has two decimal places
  e.g. Sales Taxes
- Quicker mental math
- Getting an estimate
- Handling the mismatch between fractions and decimal
  e.g. $\frac{1}{3}$ to 0.333333333333...
  - Extra: What is $0.\overline{9}$?

Procedure of Rounding

1. How many decimal places are we rounding to? Let's say we're rounding to 2 decimal places.
2. Pick the place after that one, i.e. the 3rd decimal place in our example.
3. Is the digit in that place 4 or less **or** is 5 or more?
    - ≤ 4: Do nothing to the digit before, i.e. the 2nd decimal place here
    - ≥ 5: Add 1 to the digit before, i.e. the 2nd decimal place here
        - **Note**: If the 2nd decimal place was 9 and now becomes 10, we carry 1 again, just like a carry with addition
4. Now, cut off the smaller decimal place, i.e. the 3rd decimal place
5. Done!

Example (We Do)

Round 7.496 to 2 decimal places

- We are rounding to 2 decimal places, so we pick the 3rd
- It's more than 5, so we round up
    - So, we add one to the 2nd decimal place
    - But the 2nd decimal place digit is 9, so it becomes 10 and we carry again
- To finish off, we cut off the 6 and we have 7.50

Examples (You Do)

Round the following:

1. 7.842 (1dp) = _____
2. 45.65 (1dp) = _____
3. 12.969 (2dp) = _____
4. 5.49284 (3dp) = _____
5. 0.007891 (3dp) = _____

What numbers could have been rounded to get the following?

1. _____ (2dp) = 5.82
2. _____ (1dp) = 3.5
3. _____ (2dp) = 0.70

- Homework: 5B as per your worklog (also I forgot to mention 5A)

Activity: Decimal Rounders

How to Play

- Each player chooses a hexagon without a counter.
- Each player takes turns to **roll the die 3 times**. **Add 3 to each dice number**. The results make the decimal number on the hexagon.
- The player then has to round the number to **1 decimal place**
- If the player rounds the number correctly, they can put a counter down. Otherwise, the other player does.
- The game finishes when all the hexagons are covered up. The winner of the game is the person who manages to cover up the most hexagons.

Example: If you roll a 3, 1 and 4, you add 3 to each and get 6, 4, and 7.
So you get **6.47** and round to **6.5**

5C How to add and subtract decimals

1. Line up the decimal points in the addends

```
  9.8
+ 2.12
------
```

2. Add 0s to the end if one addend has fewer decimal places: `9.8` becomes `9.80`
3. Add or subtract as normal.

```
  9.80
+ 2.12
------
 11.92
```

Note: If there is no decimal point, then you are dealing with a whole number

- Remember the decimal is to the right of the whole number, i.e. 3 = 3.0
- The same step of adding 0 to the end applies to whole numbers too.

Other Tips

- Think of decimal numbers as **dollar** amounts. This may help you remember where to line up the decimals.
- Rules for adding and subtracting are the **same** for decimals as for **whole numbers**.
- Don’t forget to put a decimal point in your answer

Examples (We Do)

a) 64.8 + 3.012

b)

```
  3
- 0.123
_______
```

c) 6.93 + 3 =

Your Turn

```
  2.3
+ 1.03
_______
```

```
  12.06
- 11.1
_______
```

```
3.236 + 1
```

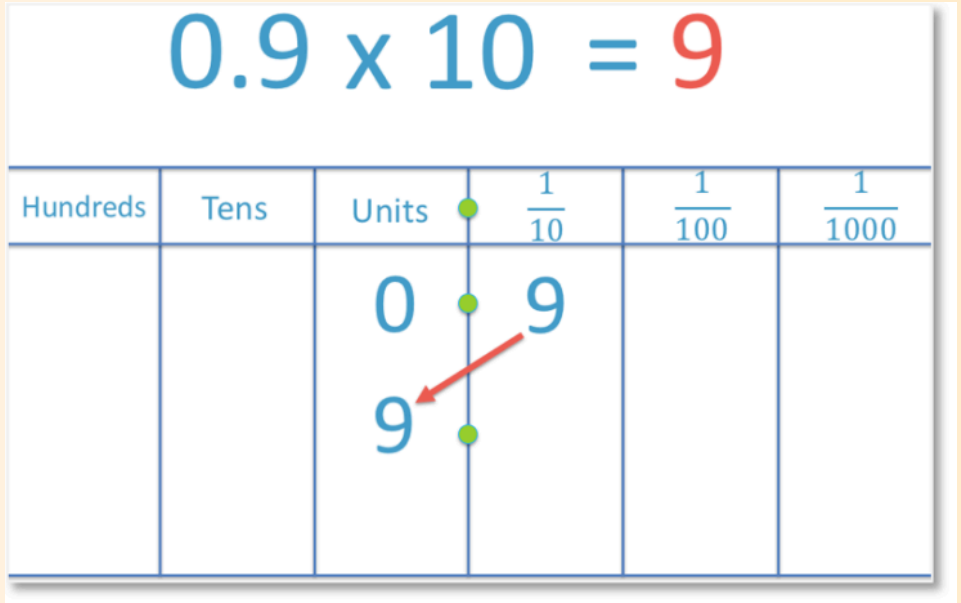
```
   1.3
 - 0.265
_______
```

```
7.5 + 6 =
```

```
4 - 0.67 =
```

5D: Multiplying and dividing decimals by 10, 100, 1000 etc.

What happens when we multiply a whole number by 10?

- When multiplying a decimal by 10, the digits move to the left by one place
- Example: 0.9×10 3.4×10 4.63×100
- 

Moving places: a trick

When you're multiplying by 10 or 100, think of how many places to move the number (1 for 10, 2 for 100, etc.)

- It's easier to look at it as if the decimal point is moving

Remember: I've drawn it like the decimal point doing a froggy jump, but it's actually the number

You Try

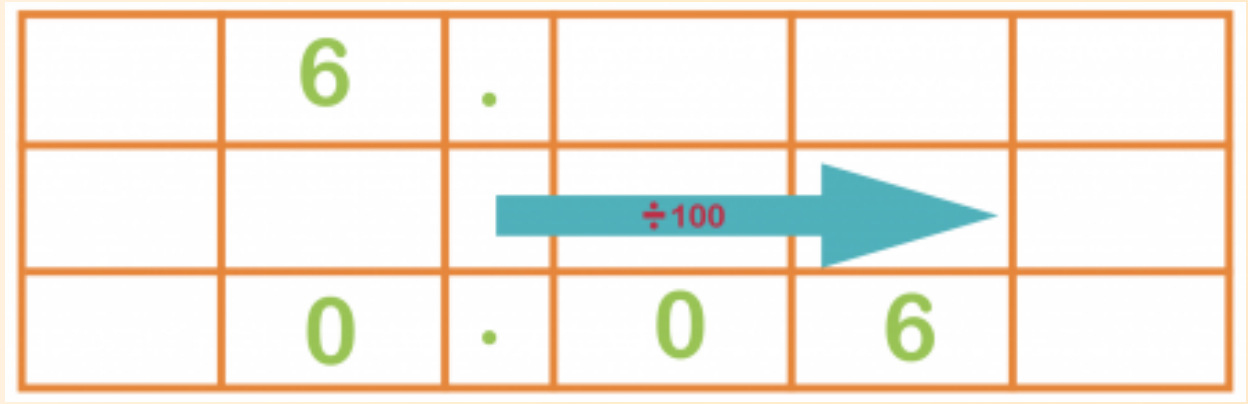
- 18×10
- 3.90×10
- 1.08×100
- 27.9001×100
- 0.056×10

Answers

- 180
- 39
- 108
- 2790.01
- 0.56

Dividing by 10, 100 and 1000

What direction will the digits move?

- When dividing by 10, the digits move to the right one place.
- Example: 6 ÷ 100 56 ÷ 10 313 ÷ 1000
- 

Your Turn

- 55 ÷ 100
- 620 ÷ 1000
- 72 ÷ 10
- 10400 ÷ 10000
- 1.26 ÷ 100

Answers

- 0.55
- 0.62
- 7.2
- 1.04
- 0.0126

Is there a word for 10, 100, 1000 and so on?

- Yes! We call them "powers of 10"

$$100 = 10 \times 10 = 10^2$$

- So right now, we're multiplying/dividing by powers of 10

Examples

- What's 10^3?
- Express 10000 as power of ten (also called index form)
- Challenge: if $10^2 = 100$, what's 10^1? What's 10^0?
- Quick way to see: count the zeroes and that's the power of 10

What if we divide or multiply by a really big power of 10?

For example, if we multiply 2.99792458 by 100000000

- this is a specific value that holds meaning in physics
- but it's also kinda a pain to work with
- physicists love rounding things, so they would use it as:
  - 300000000
- except they do something extra to make it easier to look at
- seriously, how many zeroes is that, how am I supposed to make sure I don't forget one

Introducing.... scientific notation!

We can represent 100000000 as 10^8!

- That means 300000000 can be: 3×10^8
- This form is called scientific notation

Examples of Scientific Notation

- Earth's population: approx 7.951 billion = 7.951×10^9
- Sun's mass: approx 1.988×10^{30} kg
- Stars in the Milky Way: approx 2×10^{11}

Procedure

- We have a number
  - e.g. $27,000$
- We take its first nonzero digit as the whole part, put the rest right of a decimal point
  - e.g. 2.7
- What power of 10 did we have to divide our number by to get this?
  - (hint: we can divide the original number by the decimal number)
  - e.g. $27000 \div 2.7$
- Now we take the two parts and put them together with a times sign (×)
  - 2.7×10^4
- And that's it!

Examples (You Try)

50		81,300
8.34		9,123
600,000,000		30.0
30,000,000		570

Answers

5×10^{1}		8.13×10^{4}
8.34×10^{0}		9.123×10^{3}
6×10^{8}		3.0×10^{1}
3×10^{7}		5.7×10^{2}

Homework

- 5C
- 5D

  From previous lessons:
- 5A and 5B

I will be checking on next Monday

Today's Agenda

- Multiplying by Decimals
  - Examples
- Dividing by Decimals
  - Examples
- Activity
- Worksheets

Multiplying by Decimals

What's the difference between multiplying by 8 and by 0.8?

We can represent 0.8 as $\frac{8}{10}$, so our arithmetic for 13×0.8 becomes

$$
\begin{aligned}
& 13 \times \frac{8}{10} \\
= & \frac{13}{1} \times \frac{8}{10} \\
= & \frac{13 \times 8}{1 \times 10} \\
= & \frac{104}{10} \\
= & 10.4
\end{aligned}
$$

Multiplying by Decimals, simplified

- Count how many decimal places both the multiplicands have in total
- Do the multiplication as if there are no decimal places
- Put the decimal point back so that the number of decimal places is the same as before

Tips

- If you're not sure that you have the right number of decimal places, round your multiplicands, multiply and then see if your answer is close to the rounded one, e.g. multiplying by 3.8 should be close to multiplying by 4
-

Example (We Do)

```
   4.5
 × 2.4
 _____
```

```
  1.80
+ 9.00
_______
 10.80
```

Your Turn

- 7.5 × 7.2 =
- 6.6 × 19 =
- 6.5 × 50.3 =
- 3.99 × 4.5 =
- 4.9 × 3.75 =

Answers

- 7.5 × 7.2 = **54**
- 6.6 × 19 = **125.4**
- 6.5 × 50.3 = **326.95**
- 3.99 × 4.5 = **17.955**
- 4.9 × 3.75 = **18.375**

Dividing by Decimals

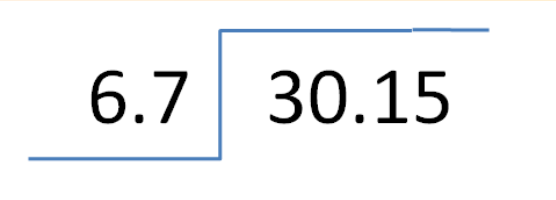
What happens when we divide by 10?

What happens when we divide by 0.1?

- It'll be like multiplying by 10, since $0.1 = \frac{1}{10}$
- So when we divide by a decimal, we do it like this:
  - Representing the operation as a fraction
    e.g. $\frac{18}{4.5}$
  - Multiply the numerator and denominator by a power of 10 so that the denominator becomes whole
    e.g. $\frac{18 \times 10}{4.5 \times 10}$
    $= \frac{180}{45}$
  - Now do the division like normal
    $\frac{180}{45} = 4$

Long Division with Decimals

We do it like normal long division, except we don't leave a remainder



Let's do this one together on the whiteboard

- Step 1: Multiply both by a power of 10 so our divisor becomes a whole number
- Step 2: Divide like normal, keeping our quotient's places aligned with the dividend's
    - Where you bring down digits that are past the decimal point, your quotient will have digits past the decimal place
- Step 3: Put the decimal point back in the quotient

Your Turn

- 1.84 ÷ 0.2 =
- 0.48 ÷ 0.06 =
- 3.366 ÷ 1.1 =
- 2.84 ÷ 7.1 =
- 0.96 ÷ 80 =

Word Problems

1. While at the grocery store, Mrs. Martin noticed that there were two different sized bottles of hot sauce, one was 360.9 grams and the other 134.67 grams. If the first one cost $4.01 and the second cost $1.34, which provided more grams of hot sauce per dollar?
2. Carpeting costs $9.99 a metre. If Jan buys 17.4 metres, how much will it cost her? (Round your answer to the nearest hundredth)
3. The basic postage at the office is $1.25 and covers any letter that is lighter than 7.5 grams. For heavier letters, extra postage of $0.87 is needed for every 10 grams over 7.5 grams. What is the postage needed for a letter that is 37.5 gram?
4. There is a rectangular lawn of dimensions 8.75m by 2.4m. If the cost of mowing the lawn is $ 54.6, what's the cost per square metre?

Decimal ÷ Snakes × Ladders

How to play

1. Get into groups of 2-4.
2. On your turn, roll the dice and advance your counter the respective number of squares.
3. If the square you land on has an operation (e.g. 0.7 ÷ 0.175), do the calculation and move your counter to the resulting decimal. All the operations should result in numbers with 1 decimal place: no rounding should be required.
    - Your counter now stays there until your next turn. I repeat, If the new square also has an operation, **don't** calculate and move again.
4. It's now the next person's turn
5. The first player to get their counter to the ≥ 10 bar wins.
   Notes: The numbers increase from 0.0 to 9.9, each row going from .0 to .9 left to right.
   If you reach the end of a row and still have to advance, go to the next row from the left,
    - e.g. if you start from 2.8 and roll a 4, you should land on 3.2

5G Connecting decimals and fractions

When the fraction's denominator is a power of ten

- Just treat it like dividing a decimal by a power of ten: shift the digits to the right of the decimal place

When the fraction's denominator is not a power of ten

- To convert a fraction to a decimal, we use long division
- We divide the numerator by the denominator

e.g. $\frac{1}{7}$

Let's do this together on the whiteboard

Mixed fractions

The long division approach also works for improper fractions.

But mixed fractions are a different story

- The whole part becomes the whole part in the decimal
  e.g. $2\frac{4}{5}$ will become:
  - 2 remains as 2
  - $\frac{4}{5}$ = 0.8
Ones	.	Tenths	Hundredths
2	.	8	

Examples (We Do)

Convert the following to decimals:

1. $3\frac{8}{13}$
2. $4\frac{5}{12}$
3. $\frac{405}{14}$

Your Turn

Convert the following to decimals

1) $2\frac{4}{8}$	2) $3\frac{5}{6}$
3) $10\frac{8}{9}$	4) $5\frac{5}{16}$
5) $3\frac{17}{24}$	6) $10\frac{55}{100}$
7) $\frac{38}{23}$	8) $\frac{47}{14}$
9) $\frac{19}{9}$	10) $\frac{50}{11}$

5H Converting Decimals to Percentages

What's a percentage?

- A fraction out of a 100

Good news: we barely have to do anything to convert decimals to percentages
Because decimals are all in base ten
So we just multiply by 100 and we're done

- If our decimal is 0.21, our percentage is
- If our decimal is 0.05, our percentage is
- If our decimal is 0.8, our percentage is
- If our decimal is 0.742, our percentage is
- If our decimal is 2.01, our percentage is

Examples (Your Turn)

Convert the following to percentages

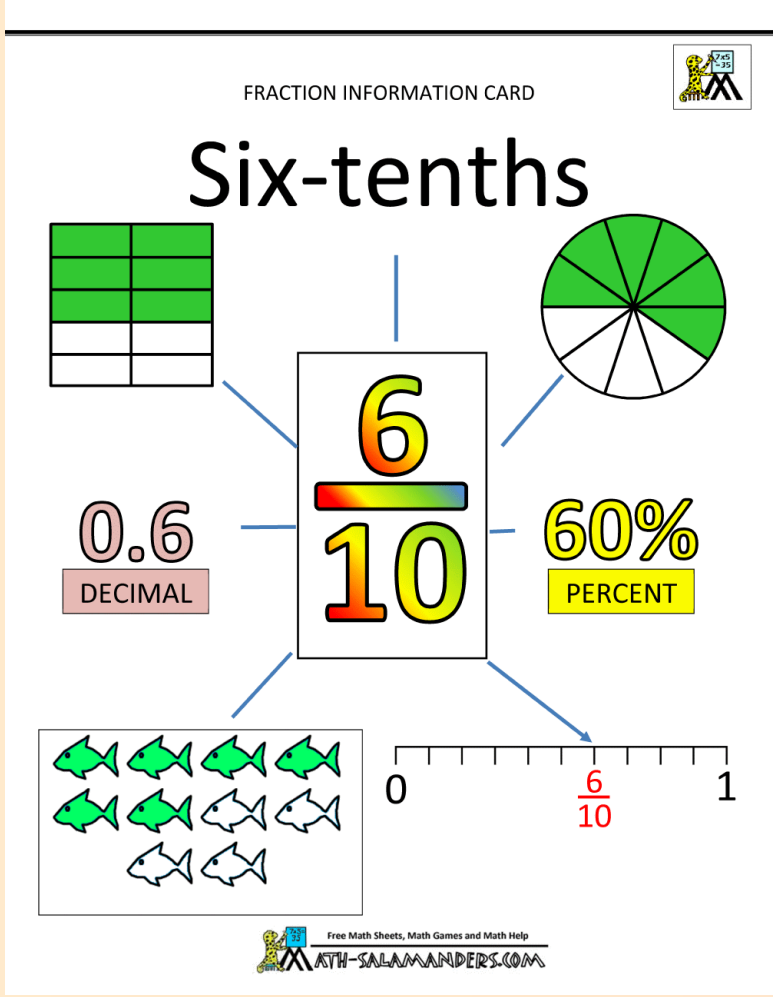
1) 0.39	2) 0.7	3) 0.88	4) 0.113	5) 0.74
6) 1.16	7) 0.143	8) 0.60	9) 2.98	10) 5.9

Convert the following to decimals

a) 89%	b) 167%	c) 71%	d) 6%	e) 88%
f) 134%	g) 58.7%	h) 53%	i) 90%	j) 15.2%

Summary

To ➡️, from ⬇️	Fraction	Decimal	Percent
Fraction		Long division	Long division then multiply by 100
Decimal	Denominator as a power of ten, then simplify		Multiply by 100
Percent	Denominator is 100, then simplify	Divide by 100	



Guess Who

Use the clues to find the correct option from the 6 below.

A) $\frac{1}{4}$	B) 60%	C) 0.9	D) 35%	E) 0.2	F) $\frac{79}{100}$

Riddle 1	Riddle 2
I am more than a half.	I am less than ¾.
I am less than nine-tenths.	I am not a fraction.
If you write me as a decimal, I have two decimal places.	I am more than 25%.
	If you round me to the nearest whole, then I round up to 1.

Learning Intentions

1. To express proportion as a fraction, decimal and percentage
2. To use fractions, decimals and percentages to compare proportions

Agenda

- Approximation and Repeating
- Expressing proportions in other forms
  - Example (We Do)
  - Your Turn
- Comparing proportions
  - Examples (We Do)
  - Your Turn
- Blooket

Approximation and Repeating

Last week we converted fractions to decimals

Some of them we rounded, because otherwise they'd repeat forever, e.g. $\frac{1}{7}$

- Instead of rounding, we can use a symbol for repeating, like so: $\frac{1}{7} = 0.\overline{142857}$
- There are other symbols, such as $\frac{1}{7} = 0.\dot{1}42857\dot{}$, but the bar is the clearest
- When there is only one repeating digit, we generally use a dot like: $\frac{1}{3} = 0.\dot{3}$

Other times, some decimals aren't exact and we round them, e.g. π

- The sign we use in this case is: $\approx$, e.g. $\pi \approx 3.14$

Examples

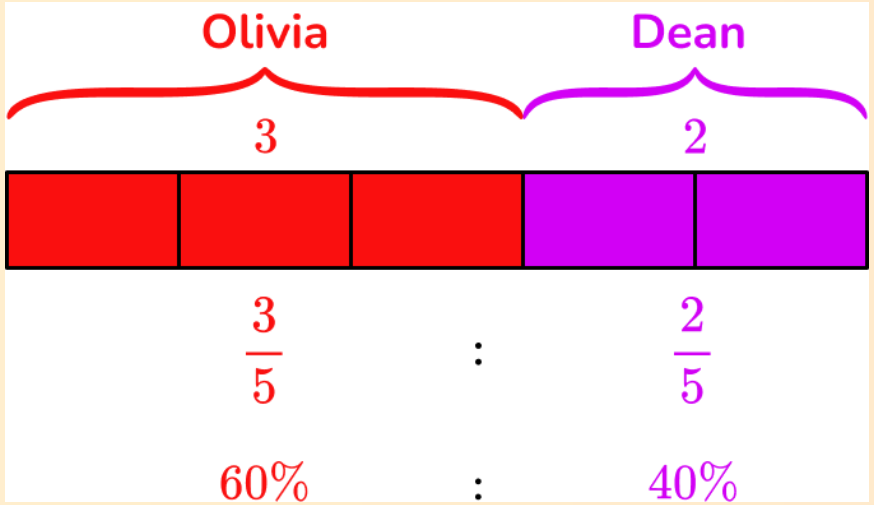
Copy down this table and fill it out, using repeating notation for each

Fraction	Decimal
$\frac{2}{7}$	
$\frac{5}{11}$	
$\frac{8}{3}$	
$\frac{4}{15}$	

5I Expressing proportions using decimals, fractions and percentages

Proportions compare one quantity to another or to a total

If you think this sounds like fractions, you're right. Proportions can be converted to fractions:



e.g. We are diluting an acid. The acid is 1 part per 90 parts of water. What percentage of the solution is acid, rounded to the 2nd decimal place?

Q: We are diluting an acid. The acid is 1 part per 90 parts of water. What percentage of the solution is acid, rounded to the 2nd decimal place?

- Total parts in the solution = parts of acid + parts of water
- Total parts = 1 + 90 = 91
- Fraction of acid in solution = parts of acid ÷ total parts = $\frac{1}{91}$
- Now, to convert to percentage, we multiply by 100 and complete the division
- We get: $\frac{100}{91}$% = 1.098% (cutting off after the third decimal place)
- After rounding, we have: percentage of acid in solution = 1.10%

Examples (Your Turn)

1. When playing a game, Anuradha scores 55 points and Jiacheng scores 77. What percentage of total points did Anuradha score?

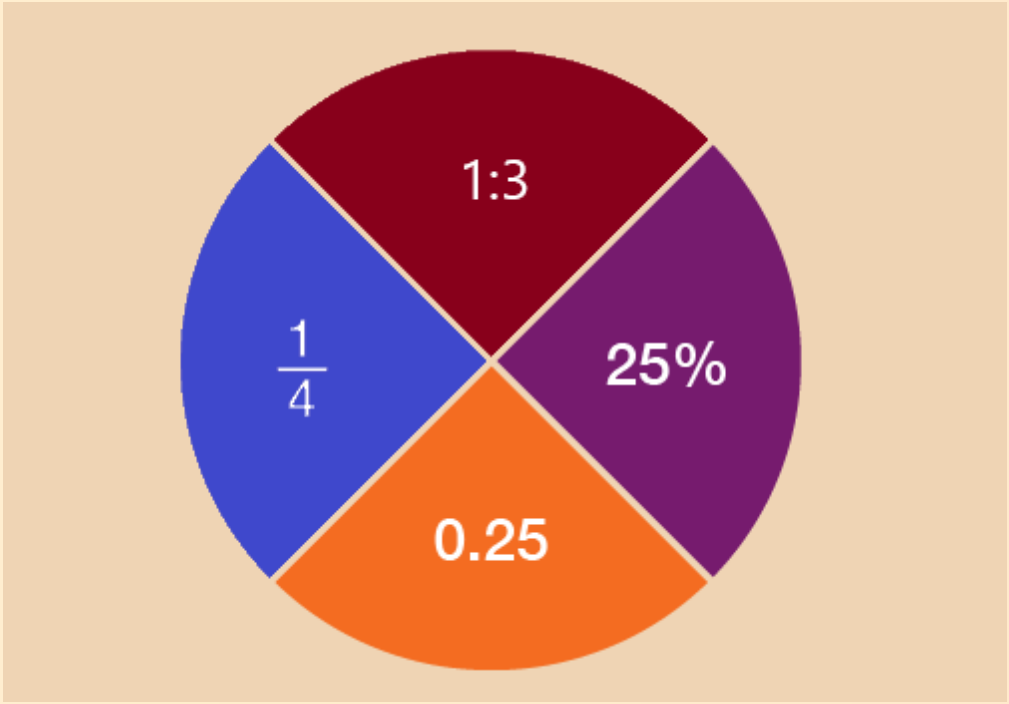
4. A pizza has 3 types of toppings on it: olives on 2 slices, pineapples on 1 slice, and pepperoni on 5 slices. No slice has more than one topping on it. Ali wants the pizza to be at least 30% olives. Will he be happy?

5. What percentage of a standard deck of playing cards is number cards, rounded to the first decimal place?

6. Ice cream is legally required to contain at least 10% milk. If I make a frozen dessert from 400 g milk, 200 g cream, 4 egg yolks (which each weigh 15g) and 80 g sugar, is it ice cream?

Comparing proportions

We convert all our values to the same type: whether percentages, fractions or decimals. Then we can sort them.



Example (We Do)

1. For every dollar Jose makes, he pays 16% in tax, he spends 27 cents on rent, he spends a quarter of it on food and saves the rest. Represent all the categories as decimals and decide which category Jose puts the least money into.

Your Turn

1. Sofia spends 59% of her free time studying, Noor has 5 hours free and spends 2.75 of those hours studying and Sasha studies twice as long as she plays video games (which are the only things she does in her free time). Convert the values to decimals and find out who spends the highest fraction of her time studying.
2. Lorenzo's scrambled eggs are 2% black pepper, $\frac{3}{4}$ eggs and the rest is cream. Charlotte adds cream in a 1:3 ratio with eggs and doesn't use black pepper. Whose scrambled eggs contains more cream?
3. Alexandria and Aleks have decided to compete to see who deserves to be called Alex more. Whoever's name has a higher percentage of vowels will win. Who wins?
4. Lottie's diet is as follows: 0.35 of a plate is covered in vegetables, 50% of her food is grains and $\frac{3}{20}$ is protein. It's recommended that $\frac{1}{2}$ of your plate be vegetables, $\frac{1}{4}$ be grains and $\frac{1}{4}$ be protein. What should she reduce and what should she increase to be healthy?