Algebra

- al-jabr is Arabic for "reunion of broken parts"
- from the 9th century to the 21st
- Pioneered by al-Khwārizmī

By the end of the first lesson you will

ALL will understand algebraic expressions from words

MOST will write algebraic expressions from words

SOME will write expressions following BIDMAS

New word: Pronumeral

A stand-in for 'some number we don't know'

- What are some examples of numbers you don't know?

Example: footy seating

Let's say that MFHS decides we need more seating to watch sports

- (this is a purely made-up scenario)
- (do not ask me to build a stadium)

How many people would it fit?
Let's say....

- *n* people
  - *n* is a pronumeral!

- What if a bench that seats 3 people gets damaged and we have to take it out?
  - *n* - *3* people

- What if instead we add in 3 more benches?
  - $n + 9$ people

- What if instead we add in 100 more benches?
  - *n + 300* people

Extension: what if we put in *b* more benches? How many could the stadium fit then?

- *n + 3b* people

Which of these expressions are equivalent?

- $n - 3$
- $3 - n$
- $n + 9$
- $9 + n$

Addition	Subtraction
$t + 5 = 5 + t$	$t - 5 \neq 5 - t$
Order doesn't matter	Order matters

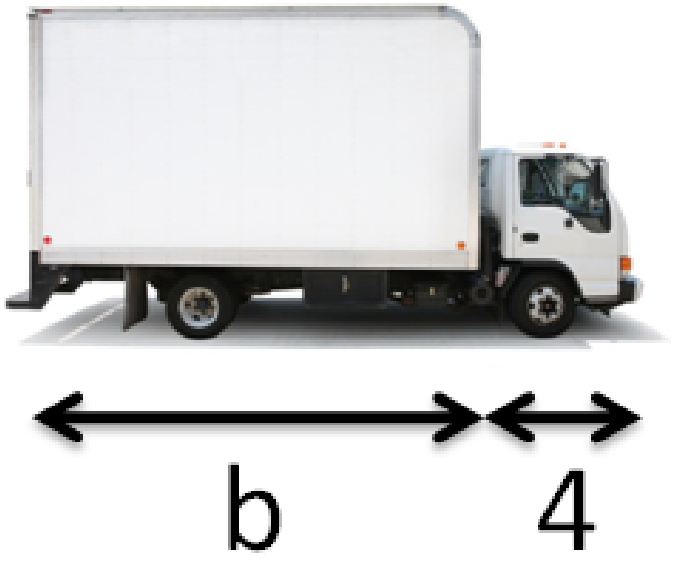
We call this the **Commutative property** of addition

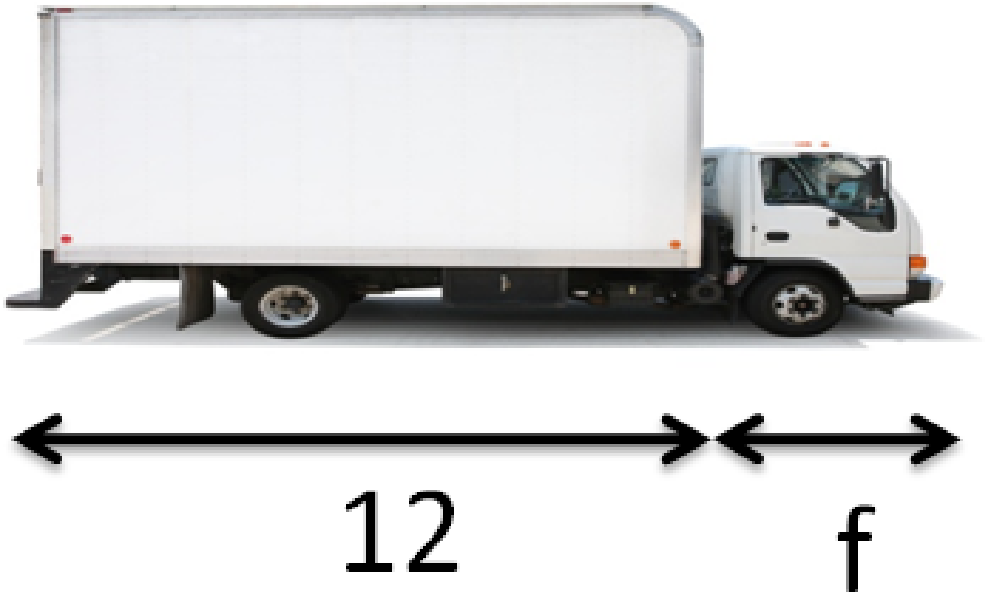
Example

If we add 4 people to a team of t people, then we have:

- $t + 4$ or $4 + t$ people on the team

Examples





Q. Find expressions for the length of these lorries and their trailers.

- b + 4 = 4 + b
- 12 + f = f + 12

Fill in the blanks

- 5 more than k is _____
- t less than 20 is _____
- 20 less than t is _____
- m more than 0 is _____
- 3 lots of n is _____

Back to footy benches

Say we have a game with Ingleburn High School and need space for their students. We magically expand the stadium to hold twice as many people. How many can the stadium fit then?

- *2n* people

We just used an abbreviation!
2n is short for 2 × n

- ~~This is because good mathematicians are lazy (but within safe limits)~~
- ~~they find easier ways to do things~~
- ~~this is called being clever and finding shortcuts~~
- ~~(as long as you're still right)~~
- ~~(otherwise it's a mistake)~~

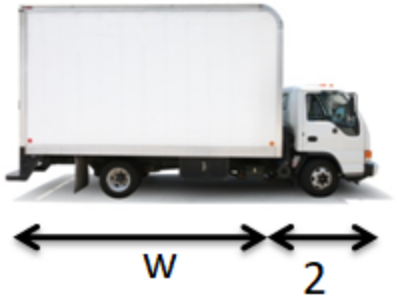
Exercises

1. add 2 to a
2. subtract 1 from b
3. double c
4. add 1 to d
5. subtract e from 3
6. ⭐ half of f

Construct an expression

Do this: make an expression for the lorry's length

1)



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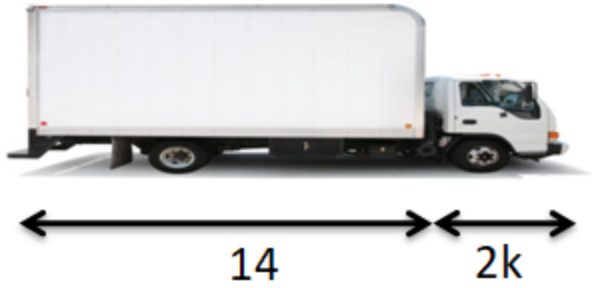

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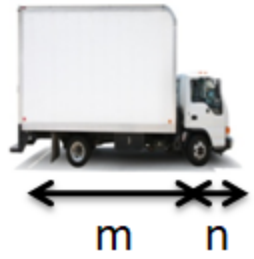
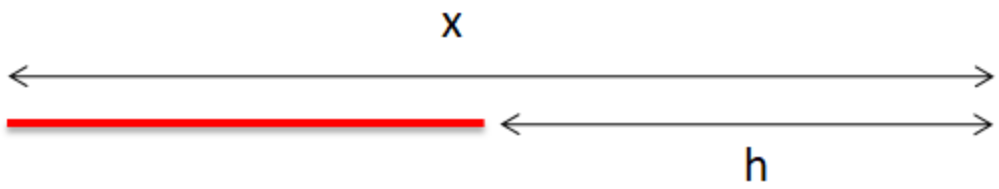


Try this: find an expression for the red length

1)

2)

3)



Write the expressions that the following describe:

GOOD		
1) 4 more than m	2) n less than h	3) 3 more than c
4) d less than 7	5) q more than a	6) 8 less than t
7) h less than p	8) k more than 5	9) f more than r

EVEN BETTER IF		
1) Multiply f by 3	2) Divide m by 5	3) Multiply g by 7
4) Half k	5) Double w	6) Multiply a by m

EXCEPTIONAL
1) Add 4 to m and then multiply the result by 5
2) Multiply p by 5 and then divide by 3
3) 4 more than the square of n
4) Subtract 3 from y and then half the result

Word problems

1. Last week Neha had x dollars. She washed cars over the weekend and now has 50 dollars. Write an expression for how much money she made washing cars.
2. Amir is baking a cake and already put in f cups of flour. If he puts in 2 more cups, how many more cups of flour are there?
3. Jianchen had 27 peaches and 11 pears left at his roadside fruit stand. He went to the orchard and picked more peaches to stock up the stand. There are now y peaches at the stand, how many did he pick?

4. There are 11 walnut trees and 26 poplar trees currently in the park. Park workers will plant *w* more walnut trees today. How many walnut trees are there now?
5. Aunty Anne is *r* years old and her brother Uncle Josh is eight years younger. How old is he?
6. Keith is *m* years old. He has a twin sister. What is her age?
7. Enrique found 74 seashells on the beach, he gave Aygun *s* of his seashells. How many seashells does he have left?
8. Giti is *g* years old. Her husband Adam is *a* years old. What is their combined age?
9. Add 14 to *n* then divide by 2.
10. ⭐ Multiply n by 2 and square it.

Star question

Omar has 2 cups of water. Akari has 5 cups of water. Peter has 1 cups of water. How many cups do they have in total?

Omar has $2x$. Akari has $5x$. Peter has x. How many x do they have in total?

Algebra mix and match

- I am handing out a piece of paper to each of you
- Each piece of paper has either an algebraic expression or a description
- Each one expression and one description match
- Find the student who has the match to yours
- We will take 5 minutes

Example

$\frac{x+y}{4}$ 🤝 "Add x and y then divide the result by 4"

Parting puzzle: Algebra magic

- Pick any number
- Add 3
- Double the result
- Subtract four
- Divide that number in half
- Subtract your original number
- What answer do you get?

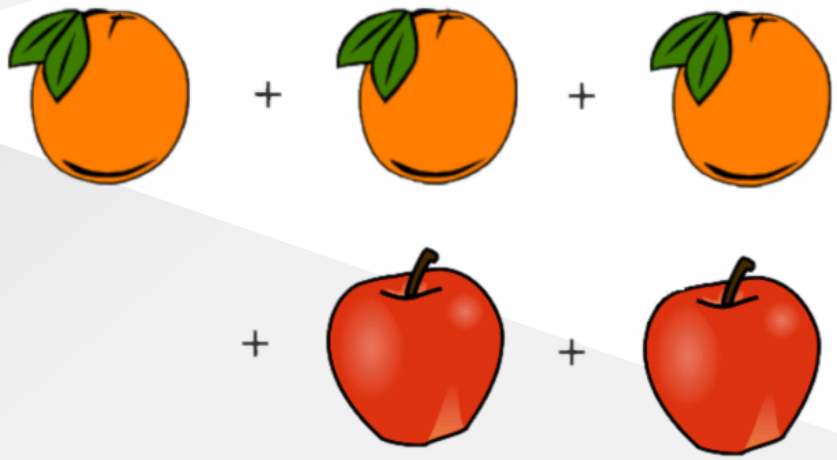
4B: Substituting into algebraic expressions

"Evaluate an expression (for a value)"	"Substitute values"

- These both mean to replace the pronumerals with a number and then calculate the result
- For example: if $a = 2$ and $b = 3$
  Then when we evaluate $a + 2b$,
  - We get $2 + 6 =$ **8**

Remember: 2x + 3 ≠ 5 or 5x

- We must substitute x before we can simplify the expression



It's like apples and oranges

Rules of Substitution

- When we do multiplication with pronumerals, we leave off the × symbol, e.g. $4a = 4 \times a$
  But, when we substitute a value, we must bring back the × symbol
  Because two digits together make a number rather than the product, e.g. 42 is not 4 × 2.
- Be careful if you subsitute a negative number in
  You will have to add brackets around it if there is another operation before it, e.g. 4 + (-5)
  For now, we will only substitute positive integers in

Procedure to substitute

First replace all pronumerals with numbers, then evaluate using order of operations (BIDMAS)
– brackets first
– then, multiplication and division from left to right
– last, addition and subtraction from left to right

Example

Simplify: $3 \times 4 + 8 - (11 + 4) \div 3$

- Brackets first: $3 \times 4 + 8 - \mathbf{(11 + 4)} \div 3 = 3 \times 4 + 8 - 15 \div 3$
- Multiplication and division: $3 \times 4 + 8 - 15 \div 3 = 12 + 8 - 5$
- Addition and subtraction: $12 + 8 - 5 = 15$

Substitution Examples

Given m = 5, evaluate:

a) m+8	**b) 9m**	**c) $\frac{15}{m}$ + 4 - m**
m + 8 = 5 + 8	9m = 9 × 5	$\frac{15}{m}$ + 4 - m = $\frac{15}{5}$ + 4 - 5
= 13	= 45	= 3 + 4 - 5
		= 2

We substituted *m* in and simplified

So, as you can see, our final answer has no pronumerals or operations

Example 5 Substituting multiple variables/pronumerals

Substitute $x = 4$ and $y = 7$ to evaluate these expressions.

a $5x + y + 8$

b $80 - (2xy + y)$

SOLUTION

a
$$\begin{aligned} 5x + y + 8 &= 5 \times x + y + 8 \\ &= 5 \times 4 + 7 + 8 \\ &= 20 + 7 + 8 \\ &= 35 \end{aligned}$$

b
$$\begin{aligned} 80 - (2xy + y) &= 80 - (2 \times x \times y + y) \\ &= 80 - (2 \times 4 \times 7 + 7) \\ &= 80 - (56 + 7) \\ &= 80 - 63 \\ &= 17 \end{aligned}$$

EXPLANATION

Insert the implied multiplication sign between 5 and x before substituting the values for x and y.

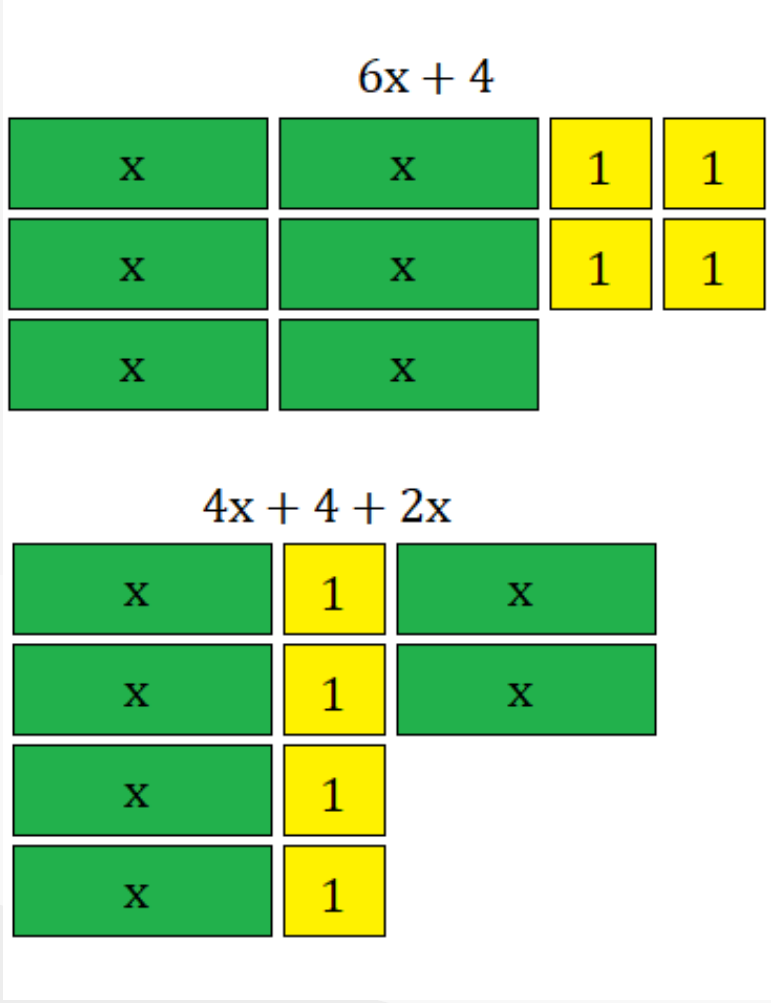
Insert the multiplication signs, and remember the order in which to evaluate.

Note that both occurrences of y are replaced with 7.

Please do Exercise 4B as per your worklog

4C: Equivalent algebraic expressions

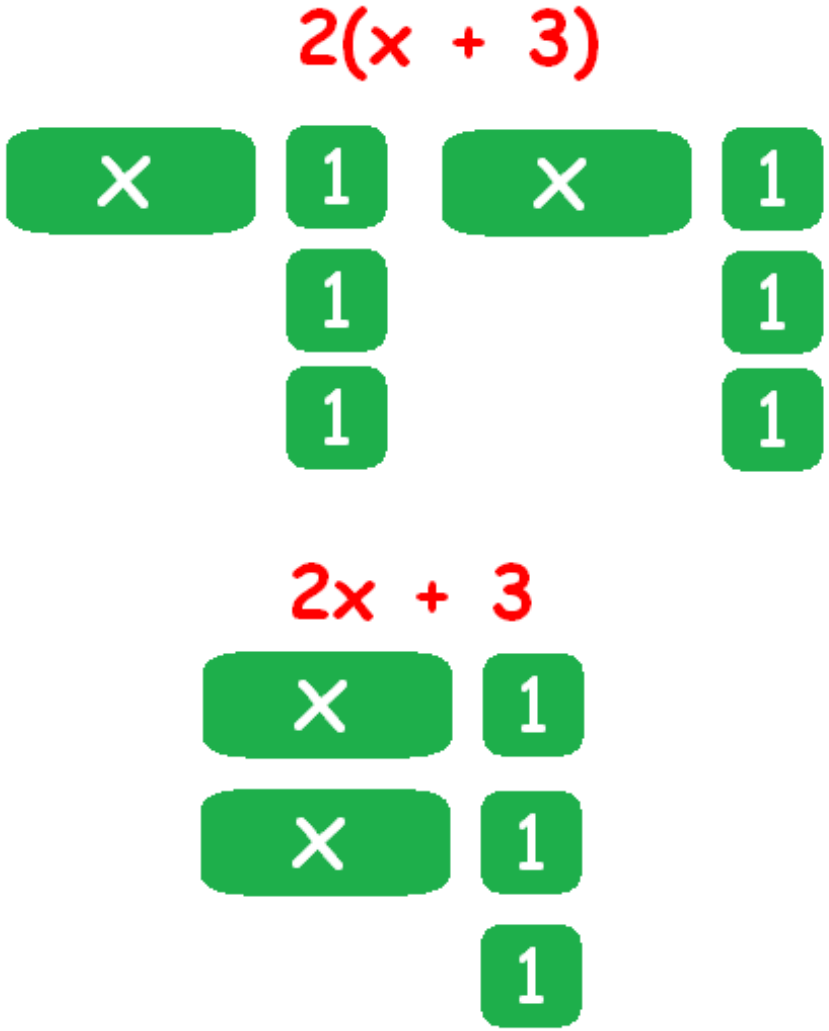
- These are expressions that have the same value, no matter what number we replace the pronumerals with
- Equivalent means 'Equal value'
- Remember equivalent fractions?
- For example, 5x + 2 is equivalent to 2 + 5x and to 1 + 5x + 1 and to x + 4x + 2
- Try It Out: Which of the following expressions are equivalent?
  3x + 4, 8 - x, 5x + 4 - 2x
  - To check, draw a table of values



Let's draw a table of values:

x =	1	2	3
3x + 4	7	10	13
8 - x	7	6	5
5x + 4 - 2x	7	10	13

- Despite their all having the same value for x = 1, we can see that only 3x + 4 and 5x + 4 - 2x have all the same values.
- So: 3x + 4 and 5x + 4 - 2x are equivalent
- Next Example: Is 2(x+3) equivalent to 2x + 3?



No, 2(x+3) = 2x + 6 instead

- This is another way to find if expressions are equivalent
  - ask me about it later if you're interested.

Please do Exercise 4C as per your worklog

4D: Like terms

- **Terms**: made up of pronumerals (letters) and numbers, but no +, - or ( ).
- **Algebraic expressions**: made up of terms and operators such as +, –, ×, ÷ and ( ).

- Examples:
  - 3a + 4b – a + 5 is an expression
  - 3a, 4b, a and 5 are the terms in the expression
- **Like terms**: terms containing exactly the same pronumerals
  - In the above expression, 3a and a are like terms because they both contain the letter symbol a and no other pronumeral
- Note: order can change in like terms, e.g. 5ab is a like term to 2ba

Combining Like Terms

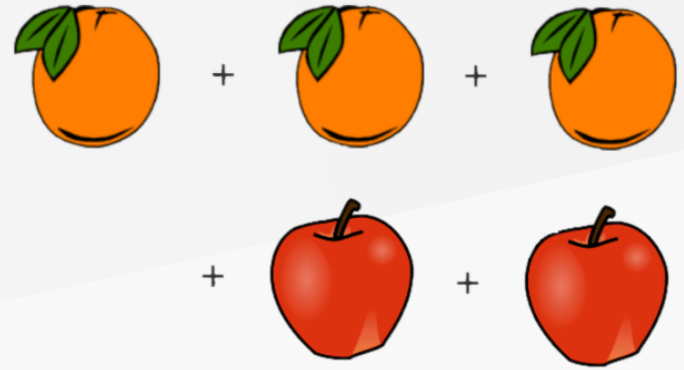
Like we combine numbers in arithmetic, we can combine like terms

$1 + 3 + 7 = ?$

$n + 3n + 7n = ?$

$4 \times 6 + 6 = ?$

$4n + n = ?$



4n, n, 3n and 7n are **like terms**.

We **collect together like terms** to **simplify** the expression.

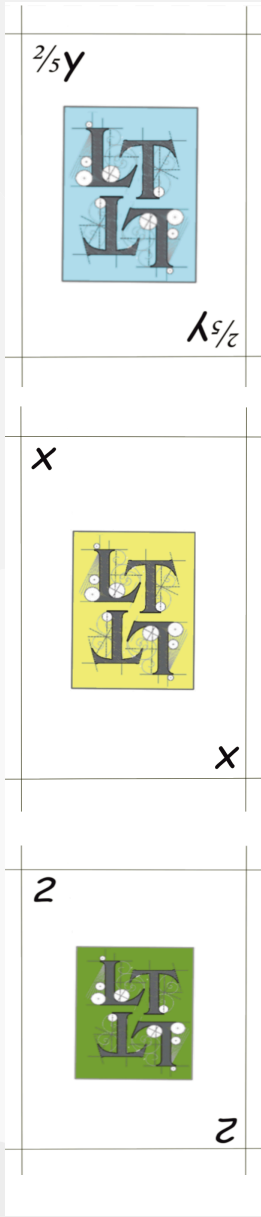
$2n + 3 + 1 + n = 3n + 4$

Remember: we cannot simplify 3n + 4 to 7n because 3n and 4 are *not* like terms!

Like Terms Uno

- Cut up these sheets of cards, make a pile and shuffle them
  Now it's like Normal Uno:
- Groups of 4: Deal out 5 cards to each player, put the rest facedown
- Flip the top card and place on the side, then play in turns
- If you can, play a card matching the colour or **like term** of the face-up card
  - Otherwise, draw a card
  - If you have two cards you can play *and* which have the same colour and are like terms, you can play both but must say what the terms combine to, e.g. you can play a yellow 5xy and a yellow -xy on the same turn
- When you have one card left, you must say "UNO"
  - Otherwise, if someone catches you, you have to pick up 2 cards

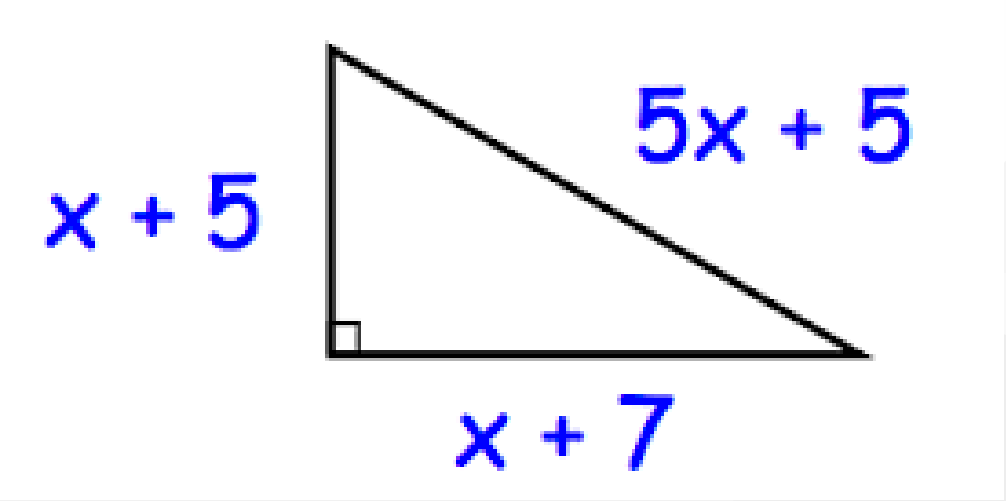
The game ends when anyone is out of cards - Call me over.
Then we'll deal 6 cards from the discard pile and everyone will simplify the term in their notebooks.

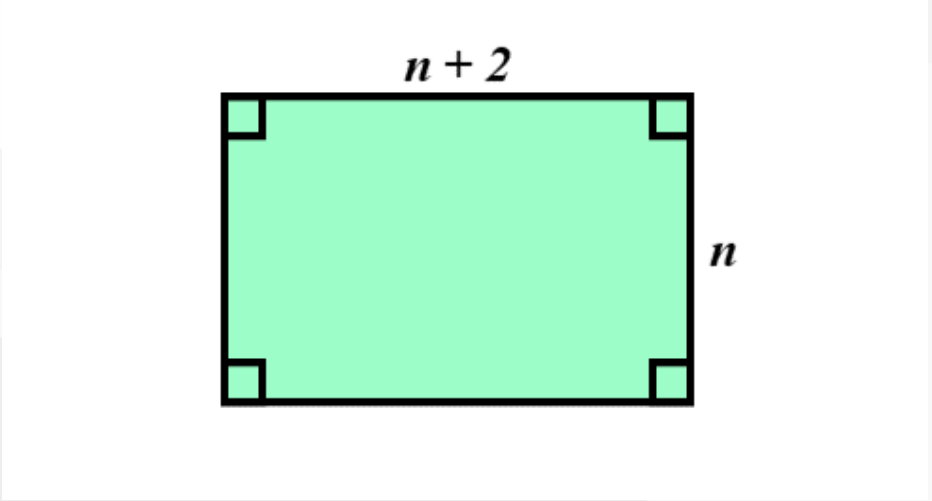


Algebraic Perimeters

Revision: How do we find the perimeter of a polygon?

- we add together the lengths of each of its sides
- Write algebraic expressions for the perimeters of the following shapes:





- More exercises

  Please do Exercise 4D as per your worklog

4E: Multiplying and Dividing expressions

Multiplying

- 3 × a is written as 3a, a × b is written as ab
- Multiplication can happen in any order so we have:
  - 3 × a is the same as a × 3 (Commutative principle)
  - 3 × (a × b) is the same as (3 × a) × b: we write these as 3ab (Associative Principle)
    - Let's Try: replace a and b by different numbers and see how these are true
- a × a is written as a^2
- We write numbers first and then letters in alphabetical order in a term, e.g. b × 2 × a is written as 2ab

Multiplying and Dividing expressions

Dividing

- k ÷ 3 is written as $\frac{k}{3}$
- We can cancel common factors in the numerator and denominator
  - e.g. $\frac{{}^{2}\cancel{4}a^{1}\cancel{b}}{{}_{1}\cancel{2}{}^{1}\cancel{b}c} = \frac{2a}{c}$

Examples of Multiplying

- $4 \times a =$
- $1 \times b =$
  - We don’t need to write a 1 in front of the letter
- $b \times 5 =$
  - We write numbers first
- $3 \times d \times c =$
  - We write letters in alphabetical order.
- $6 \times e \times e =$

Examples of Dividing

- (a + b) ÷ c =
- 25a ÷ 5b =
- 10xy ÷ 4y =
- Write (3x + 1) ÷ 5
  - Explanation: The brackets are no longer required as the fraction line makes it clear all of 3x + 1 is divided by 5

- Simplify: $\frac{8ab}{12b}$
  - Tip: Insert multiplication signs to help find common factors

Please do Exercise 4E as per your worklog

What did we do last time?

k × 3	*3k*	For multiplication, we leave off the × symbol
k ÷ 3	$\frac{k}{3}$	Division can be represented as a fraction
k × k	k^2	powers for pronumerals are represented the same as for numbers
1k	*k*	Think how 1 × 4 = 4
c × a × 3	*3ac*	Write the number first, then variables in alphabetical order

- And remember to simplify fractions!
- Now try this

$$(3a)^2 - 4a^2 + a \div a - 5a \times a - 1$$

What We're Learning Today

- Expanding Brackets
  - Combine coefficients (positive and negative)
  - Use the distributive law

4F: Expanding brackets

Multiply the “Insiders” by the Multiplier-Outside

$3(\mathbf{x} + \mathbf{2})$ ⬅ x and 2 are the **insiders**

$= 3 \times (\mathbf{x} + \mathbf{2})$

$= 3 \times \mathbf{x} + 3 \times \mathbf{2}$ ⬅ the × *distributes* over the +

$= 3x + 6$

Visualising Distribution

Examples

2 (3x + 5)

= 2 x 3x + 2 x 5

= 6x + 10

3(2x - 4)

= 3 x 2x - 3 x 4

= 6x - 12

-5(4x - 1)

= -5 x 4x - (-5) x 1

= -20x - (-5)

= -20x + 5

Now You Try

I've given you a bunch of worksheets, try however many you get to

This is the order of difficulty

- Expanding Brackets Corbett Maths (▶ icon)
- Expanding Brackets Maze
- Expanding pairs of brackets - Extension
  DoingMaths.co.uk
- Increasingly Difficulty Brackets
- Expanding **Two** Brackets Corbett Maths (▶ icon)
- Extra Examples

Expanding Brackets Bingo

Choose 9 of these answers to put in your grid

$a^2 + 9a$	$9a + 36$	$6a + 6b$	$-15a + 40b$
$-5a - 5b$	$20a + 10$	$64a + 72$	$10a - 45b$
$a^2 + 2a$	$-9a + 9b$	$-6a + 6b$	$-a^2 + 6a$
$5a - 5b$	$5a + 5b$	$-4a + 4b$	$9a - 27$

Next Question

Finish

A story

In the 1780s, a German schoolmaster told his class to add up the first 100 integers. He meant to keep the students quiet for half an hour, but one young pupil almost immediately produced an answer.

How did the student do it?

Triangle Numbers

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4G Applying Algebra

Lesson Intention: develop problem-solving skills by using algebra and equations to model and solve word problems.

1. We uses algebra to "model" a real-world situation.
    - We take the information in the given problem
    - We represent quantities using pronumerals
    - We write an expression with the pronumerals
    - The expression is our model
2. We use algebraic models to solve problems
3. Profit!

Constructing expressions from problem statements

- We see from the problem what values can change (these become the pronumerals) and which stay the same (these become the constants)

Example: A plumber charges a $60 call-out fee plus $50 per hour.

- The call-out fee: constant
- The number of hours: variable

What's the cost of an 8 hour visit? An x hour visit?

- cost = 60 + 50x

Algebraic relationships

Sometimes there are two values that change together. We can use algebra to represent the relationship between them.

Example: Amir is baking a cake. The recipe will serve 12 people and calls for 2 cups of flour. Assuming he has the rest of the ingredients, how many people can he serve if he has:

a) 1 cup of flour
b) 3 cups of flour
c) 6 cups of flour
d) x cups of flour

What's the relationship between how much flour Amir has and how many people he can serve?

Finding the unknown

Another way to decide what should be represented by a pronumeral is what quantity is unknown.

Example: Tickets for the football games were $2.50 for general admission and 50 cents for kids. If there were six times as many general admissions sold as there were kids' tickets, and total receipts were $7750, how many of each type of ticket were sold?

- Let number of kids' tickets be x
- General admissions sold = 6x
- Money from kids' tickets = $0.5x
- Money from General admissions = $2.5 × 6x
- Total sales = $15x + $0.5x = 7750
  x = 7750/15.5 = 500
- Kids' tickets: 500 sold
  General admissions: 3000 sold

Challenge

Grandma had made pies for a bake sale. She had carefully put equal amounts of mixture in each pie tin and was now trying to find the weight of the pies.

She had a problem; she only had one 250 gram weight and one 125 gram weight and a balance scale.

She found that one pie on one side of the scale was balanced with both weights and a quarter of a pie on the opposite side.

How heavy was each pie?

- Answer: 500 grams
- Working: $250 + 125 + \frac{x}{4} = x$
  $375 = \frac{3x}{4}$
  $500 = x$

Further material

Online exercises

1. The larger of two numbers is twice the smaller, and the sum of the numbers is 216. What is the smaller number?

216

Working: