

Factoring!

Breaking expressions into parts

Opposite of Expanding

Let's start with HCF

What's an HCF??

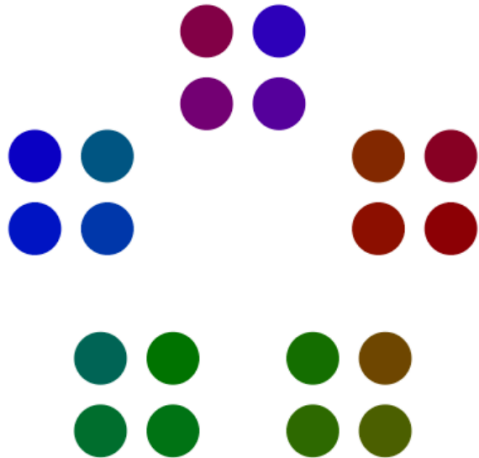
- **Highest Common Factor**

Let's break each of these words down

Watch numbers dance

- An animation of prime factors into patterns
(In maths we like patterns)

20
 $5 \times 2 \times 2$



8
 $2 \times 2 \times 2$



- What's the HCF of 20 and 8?
 - How do we get the HCF from the prime factors?

What does this have to do with Factoring?

Right now we're factoring expressions like $20x + 12$

- What's the HCF of 20 and 12?
- So we'll take 4 common
- Then we have:
 - $20x + 12 = 4(5x + 3)$
- And that's our factored expression!

Examples: We Do

- $15x + 5$
- $8x + 20$
- $16x + 10$
- $6x + 18$
- $33 + 110y$

Examples: You Try

- $18x + 9$
- $8 + 28q$
- $20q + 25$
- $18r + 9d$

Can a pronumeral be a common factor?

- Yes!
- Just as if a pronumeral were a prime
- Example: x^2+7x
 $= x(x + 7)$

Examples

$$2x^2 + 4$$

$$x^2 + 5x$$

$$9x^2 - 3x$$

$$10x^2 + 25x$$

$$6x^2 - 18$$

Extra: The Euclidean Algorithm!

- Let's say we have to find the HCF of two numbers a and b
- Divide the larger number (say, a), by the smaller (say, b)
- If the remainder (let's call it c) is 0, b is the HCF, we're done!
- Otherwise, we replace a with b and b with c and start again
 - i.e. we divide b by c
- If the remainder is 0, the last divisor is the HCF
- If the remainder is 1, a and b are coprime

Euclid's Division Algorithm Formula



$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\begin{array}{r} \begin{array}{r} 2 \\ 132 \overline{) 320} \\ \underline{- 264} \\ 56 \end{array} \longrightarrow 320 = 132(2) + 56 \\ \begin{array}{r} 2 \\ 56 \overline{) 132} \\ \underline{- 112} \\ 20 \end{array} \longrightarrow 132 = 56(2) + 20 \\ \begin{array}{r} 2 \\ 20 \overline{) 56} \\ \underline{- 40} \\ 16 \end{array} \longrightarrow 56 = 20(2) + 16 \\ \begin{array}{r} 1 \\ 16 \overline{) 20} \\ \underline{- 16} \\ 4 \end{array} \longrightarrow 20 = 16(1) + 4 \\ \begin{array}{r} 4 \\ 4 \overline{) 16} \\ \underline{- 16} \\ 0 \end{array} \longrightarrow 16 = 4(4) + 0 \end{array}$$

H.C.F H.C.F

Adding And Subtracting Algebraic Fractions

- How do we add and subtract normal fractions?
 - Step 1: Getting a common denominator
 - Find the lowest common multiple
- Same applies to algebraic fractions

Example 1

$$\frac{5}{2a^2} + \frac{c}{6ab}$$

$$\text{LCM} = 6a^2b$$

So our fraction becomes

$$\begin{aligned} &= \frac{5 \times 3b}{6a^2b} + \frac{c \times 2a}{6a^2b} \\ &= \frac{15b + 2ac}{6a^2b} \end{aligned}$$

Can we simplify any further?

Example 2

$$\begin{aligned}& \frac{3}{x+4} - \frac{5}{2x} \\&= \frac{3 \times 2x}{(x+4) \times 2x} - \frac{5 \times (x+4)}{2x \times (x+4)} \\&= \frac{6x}{2x(x+4)} - \frac{5x+20}{2x(x+4)} \\&= \frac{6x - (5x+20)}{2x(x+4)} \\&= \frac{6x - 5x - 20}{2x(x+4)} \\&= \frac{x-20}{2x(x+4)}\end{aligned}$$

or $\frac{x-20}{2x^2+8x}$

Example 3 (Your Turn)

$$\begin{aligned} & \frac{3x + 2}{2} - \frac{x - 1}{4} \\ &= \frac{(3x + 2) \times 2}{2 \times 2} - \frac{x - 1}{4} \\ &= \frac{6x + 4}{4} - \frac{x - 1}{4} \\ &= \frac{6x + 4 - (x - 1)}{4} \\ &= \frac{6x + 4 - x + 1}{4} \\ &= \frac{5x + 5}{4} \\ &= \frac{5(x + 1)}{4} \end{aligned}$$

Example 4 (You Try First)

$$\begin{aligned} & \frac{1}{2x} + \frac{4}{x+2} \\ &= \frac{1 \times (x+2)}{2x \times (x+2)} + \frac{4 \times 2x}{(x+2) \times 2x} \\ &= \frac{x+2}{2x(x+2)} + \frac{8x}{2x(x+2)} \\ &= \frac{x+2+8x}{2x(x+2)} \\ &= \frac{9x+2}{2x(x+2)} \end{aligned}$$

